

# Lorentz Violating $p$ -form Gauge Theories in Superspace

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**ABSTRACT:** Very special relativity (VSR) keeps the main features of special relativity but breaks rotational invariance due to an intrinsic preferred direction. We study the VSR modified extended BRST and anti-BRST symmetry of the Batalin-Vilkovisky (BV) actions corresponding to the  $p = 1, 2, 3$ -form gauge theories. Within VSR framework, we discuss the extended BRST invariant and extended BRST and anti-BRST invariant superspace formulations for these BV actions. Here we observe that the VSR modified extended BRST invariant BV actions corresponding to the  $p = 1, 2, 3$ -form gauge theories can be written manifestly covariant manner in a superspace with one Grassmann coordinate. Moreover, two Grassmann coordinates are required to describe the VSR modified extended BRST and extended anti-BRST invariant BV actions in a superspace. These results are consistent with the Lorentz invariant (special relativity) formulation.

**KEYWORDS:** Very Special Relativity,  $p$ -form gauge theory, Superspace.

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## 1 Overview and motivation

The standard model, although phenomenologically successful, is unable to explain a variety of issues satisfactorily [1]. The standard model is assumed to be an effective description that works in the low-energy limit of a more fundamental theory (having a quantum description of gravitation also). However, the natural scale for a fundamental theory including gravity is governed by the Planck mass. This leads to an interesting question that whether any aspects of this underlying theory could be revealed through the definite experiment with present techniques. To understand this properly, one possibility may be to examine

proposed fundamental theories for effects that are qualitatively different from standard-model physics. In this regard, one possibility is that the new physics involves a violation of Lorentz symmetry. In this connection, Cohen and Glashow proposed that the laws of physics need not be invariant under the full Lorentz group but rather under its proper subgroup [2]. An advantage of this hypothesis is that while Lorentz symmetry is violated, the theory still follows the basic postulates of special relativity, like the constancy of the velocity of light. Any scheme with proper Lorentz subgroups along with translations is referred to as very special relativity (VSR).

As an observable consequences of VSR, a novel mechanism for neutrino masses without introducing new particles has been studied in Ref. [3]. Few other observable consequences of VSR have also been given in [4, 5]. In recent past, VSR has been studied in various contexts. For instance, the idea of VSR is implemented to de Sitter spacetime where breaking of de Sitter invariance arises in two different ways [6]. Also, it has been shown that the gauge field in quantum gauge perspective acquire mass naturally without the conventional Higgs mechanism [7]. The modifications due to VSR have also been analyzed for the reducible gauge theories using the BV formulation [8]. Within VSR framework, the event space underlying the dark matter and the dark gauge fields supports the algebraic structure [9]. The VSR effect to curved space-times shows that the  $SIM(2)$  symmetry, which leaves the preferred null direction invariant, does not provide the complete couplings to the gravitational background [10]. The proper Lorentz subgroups together with translations are realized in the non-commutative space-time where the behavior of non-commutativity parameter  $\theta^{\mu\nu}$  is found lightlike [11, 12]. In VSR scenario,  $N = 1$  SUSY gauge theories contain two conserved supercharges rather than the usual four [13]. The effects of quantum correction to VSR is studied to produce a curved space-time with a cosmological constant [14], where it is shown that the symmetry group  $ISIM(2)$  does admit a 2-parameter family of continuous deformations. Recently, a violation of Lorentz invariance in quantum electrodynamics induced by a very high frequency background wave is studied, where averaging observables over the rapid field oscillations provides an effective theory [15]. The quantum electrodynamics and the massive spin-1 particle are discussed in VSR in Refs. [16, 17]. The spontaneous symmetry-breaking mechanism to give a flavor-dependent VSR mass to the gauge bosons is studied in [18]. Interestingly, a quantum field theoretic structure suitable to describe the dark matter suggest that VSR plays the same role for the dark matter fields as special relativity does for the standard model fields [9].

On the other hands, the supersymmetric version [19] of a non-Abelian gauge theory is a super-geometrical theory of a constrained super 1-form [20]. It is well-known that in a superspace formulation there exists a superconnection which is the gauge superfield in a superspace. This, eventually, extends the relation of 1-forms and gauge theories in ordinary spacetime to the superspace. As a result, the entire formalism of differential geometry is valid in the superspace approach. The higher order  $p$ -forms theories have been studied [20], which contain the gauge superfields characterized by gauge parameters of  $(p-1)$ -forms. The importance of such formulation lies to the fact that the well-known formulation of simple supergravity in eleven dimensions [21] explicitly contains a 3-form component gauge field and its extension to superspace [22] naturally includes the introduction of a super 3-form

gauge superfield. A simple reduction of eleven dimensions to four-dimensions then leads to 3-form gauge superfield in the  $N = 8$  supergravity. Also, antisymmetric tensor fields describe the low energy excitations in string theories [23, 24]. The study of higher-form gauge theory is also important for the classical string theories [25], vortex motion in an irrotational, incompressible fluid [26, 27], dual formulation of the Abelian Higgs model [28, 29] and for supergravity multiplets [30].

To quantize the  $p$ -form gauge theories, BV formulation is one of the most general and powerful approaches [31–37]. One of the important illustrations of this formulation is that it provides a systematic way of accretion of the nontrivial ghost for ghost structure for the case of reducible gauge theories. It has been observed that the antifields of the BV formulation coincide with antighosts of certain collective fields, which ensure that Schwinger-Dyson equations are satisfied as a consequence of the gauge symmetry algebra [38, 39]. Also the quantum corrections for anomalous gauge theories can be evaluated from the functional measure as long as a suitable regularization procedure is introduced [40]. A superspace formalism for the Lorentz invariant BV action of 1-form, 2-form and 3-form gauge theories have been studied [41–43]. The extended BRST and extended anti-BRST invariant formulations (including some shift symmetry) of the Lorentz invariant BV action have also been studied [41, 42, 44], which lead to the proper identification of the antifields through equations of motion of auxiliary field variables. The importance of shift symmetries introduced through collective fields lies to the fact that it ensure that Schwinger-Dyson equations at the level of the BRST algebra can be performed within the Feynman path integral [38]. According to the field redefinition theorem, the particular choice of variables should have no influence on physical quantities like S-matrix elements, which appreciates to formulate the quantization prescription in a more coordinate-independent manner. In Refs. [38, 39], Schwinger-Dyson equations is accomplished in different field variables following shift symmetry. Although VSR has been studied in various contexts, but the extended BRST and extended anti-BRST invariant formulations with their superspace description remain unstudied in VSR framework. This provides us an opportunity to bridge this gap.

We consider a non-Abelian 1-form gauge theory in VSR context which remains invariant under a non-local (Lorentz breaking) gauge transformation. The equations of motion in VSR-type Lorentz gauge leads a Proca type equation which confirms that non-Abelian vector to have a mass. Although this describes a theory with mass but remains invariant under a (VSR modified) gauge transformation. This leads to a redundancy in gauge degrees of freedom if we quantize it without fixing a gauge. Therefore, utilizing Faddeev-Popov procedure, we construct an effective action which admits a (VSR modified) BRST transformation. Furthermore, we study the extended BRST symmetry which includes a shift symmetry. This extra symmetry is then gauge fixed (adding new ghosts, antighosts, and auxiliary fields) in such a way that the original action is recovered after the extra fields are integrated out. In order to recover original theory (or to compensate these additional fields), we further introduce anti-ghosts with exactly opposite ghost numbers. Within formulation, these anti-ghosts coincide with the antifields of the BV formulation analogous to Lorentz invariant case. We further provide a superspace description of VSR modified non-Abelian 1-form gauge theory possessing extended BRST symmetry with the help of

coordinates  $(x_\mu, \theta)$ . The superspace description of this theory having extended anti-BRST invariance only needs another fermionic variable  $\bar{\theta}$  together with  $x_\mu$ . However, we found that even in VSR modified theory possessing both the extended BRST and extended anti-BRST invariance, one needs a superspace with two grassmann parameters  $\theta, \bar{\theta}$  together with  $x_\mu$  to provide a superfields description. We generalize the results of non-Abelian 1-form gauge theory to the higher-form (for instance 2, 3-form) gauge theories also to show consistency of results.

The paper is organized as following. In section II, we outline non-Abelian 1-form gauge theory in VSR. Here, we study the VSR modified extended BRST and extended anti-BRST transformations (which include a shift symmetry) for the BV action of theory. Within this section, we demonstrate a superspace description for the 1-form gauge theory having extended BRST invariance and extended anti-BRST invariance as the separate cases. Section III is devoted to the generalization of results for the non-Abelian 1-form gauge theory to the Abelian 2-form gauge theory. We further generalize these results to the Abelian 3-form gauge theory case in section IV. The paper is summarize with future remarks in the last section. The lengthy calculations are reported in Appendix sections.

## 2 Non-Abelian 1-form gauge theory: VSR modified BV action in superspace

In this section, we describe the VSR modified BV action for non-Abelian 1-form gauge (Yang-Mills) theory in superspace. Let us review first the VSR description of non-Abelian 1-form gauge theory following Ref. [18]. We start by defining the classical Lagrangian density in VSR as follows [18]

$$\mathcal{L}_0 = -\frac{1}{4} \text{Tr} \left[ \tilde{F}^{a\mu\nu} \tilde{F}_{\mu\nu}^a \right], \quad (2.1)$$

where field-strength tensor  $\tilde{F}_{\mu\nu}$  is given by

$$\tilde{F}_{\mu\nu} = F_{\mu\nu} - \frac{1}{2} m^2 \left( n_\nu \frac{1}{(n \cdot D)^2} n^\alpha F_{\mu\alpha} - n_\mu \frac{1}{(n \cdot D)^2} n^\alpha F_{\nu\alpha} \right), \quad (2.2)$$

with

$$\begin{aligned} F_{\mu\nu} = & \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu] + \frac{1}{2} m^2 n_\nu \left( \frac{1}{(n \cdot \partial)^2} \partial_\mu (n \cdot A) \right) \\ & - \frac{1}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)^2} \partial_\nu (n \cdot A) \right) - \frac{i}{2} m^2 \left[ \frac{1}{(n \cdot \partial)^2} n \cdot A, (n_\mu A_\nu - n_\nu A_\mu) \right]. \end{aligned} \quad (2.3)$$

The vector  $n_\mu$  is a constant null vector (i.e.  $n^2 = 0$ ), which transforms under a VSR transformation so that any term containing ratios involving  $n_\mu$  are invariant. The covariant derivative is determined by imposing the proper transformation property for the covariant derivative as follows:  $D_\mu = \partial_\mu - i[A_\mu, \cdot] - \frac{i}{2} m^2 n_\mu \left[ \left( \frac{1}{(n \cdot \partial)^2} n \cdot A \right), \cdot \right]$ . We use following definition to handle the non-local terms [18]

$$\frac{1}{n \cdot D} = \int_0^\infty db e^{-bn \cdot D}. \quad (2.4)$$

The equations of motion for the vector field satisfying VSR type Lorentz gauge condition leads to a Proca equation which suggest that vector field has mass [18].

This Lagrangian is also invariant under a VSR modified (non-local) gauge transformation

$$\begin{aligned}\delta A_\mu &= \partial_\mu \lambda - i[A_\mu, \lambda] + \frac{i}{2} m^2 n_\mu \left( \lambda, \frac{1}{(n \cdot \partial)^2} n \cdot A \right) \\ &\quad - \frac{1}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)} \lambda \right) + \frac{i}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)^2} n \cdot [A, \lambda] \right),\end{aligned}\quad (2.5)$$

where  $\lambda$  is an infinitesimal parameter. Being a (VSR modified) gauge invariant, the non-Abelian 1-form gauge theory contains redundant degrees of freedom. To quantize the theory correctly we need to choose a gauge appropriately. In this context, the gauge-fixed Lagrangian density together with the ghost term is given by

$$\begin{aligned}\mathcal{L}_{gf} &= \text{Tr} \left[ B \partial^\mu A_\mu - B \frac{m^2}{n \cdot \partial} n^\mu A_\mu + i \tilde{C} (\square - m^2) C + \tilde{C} \partial_\mu [A^\mu, C] + \frac{m^2}{2} \tilde{C} \left( \frac{1}{n \cdot \partial} [n \cdot A, C] \right) \right. \\ &\quad \left. - \frac{m^2}{2} \tilde{C} \left( n \cdot \partial \left[ C, \frac{n \cdot A}{(n \cdot \partial)^2} \right] \right) \right],\end{aligned}\quad (2.6)$$

where  $B^a$  is an auxiliary field. Here we note that, in comparison to VSR modified covariant gauges, the gauge-fixed Lagrangian density takes simplest form [8] in VSR modified light-cone gauge,  $\eta \cdot A = 0$ , where  $\eta_\mu$  is an arbitrary constant vector that defines a preferred axis in space. Now, the effective quantum Lagrangian density for Yang-Mills theory in VSR is given by

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{gf}. \quad (2.7)$$

This Lagrangian density leads to following vector field and ghost propagators respectively:

$$\begin{aligned}\Delta^{\mu\nu} &= \frac{1}{p^2 + m^2} \left[ \eta^{\mu\nu} - \left( \frac{\alpha - 1}{2\alpha - 1} \right) \frac{1}{p^2 + m^2} \left( p^\mu p^\nu + \frac{1}{2} m^2 (n^\mu p^\nu + n^\nu p^\mu) \frac{1}{n \cdot p} + \frac{1}{4} m^4 \frac{n^\mu n^\nu}{(n \cdot p)^2} \right) \right], \\ \Delta_{gh} &= -\frac{1}{p^2 + m^2}.\end{aligned}\quad (2.8)$$

This implies clearly that both the gauge field and ghost field have same mass  $m$ , consequently, this mass generation is different from Higgs mechanism. This is matter of calculation only to show that this effective Lagrangian density (2.7) is invariant under the following BRST transformations:

$$\begin{aligned}s_b A_\mu &= \partial_\mu C - \frac{m^2}{n \cdot \partial} n_\mu C - i[A_\mu, C] + i \frac{m^2}{2} \left[ C, \frac{n \cdot A}{(n \cdot \partial)^2} \right] + i \frac{m^2}{2} n_\mu \left( \frac{1}{(n \cdot \partial)^2} n \cdot [A, C] \right), \\ &=: \mathcal{D}_\mu C, \\ s_b C &= i C^2, \quad s_b \tilde{C} = i B, \quad s_b B = 0.\end{aligned}\quad (2.9)$$

This BRST transformations are nilpotent in nature, i.e.,  $s_b^2 = 0$ . Since the gauge fixing and ghost part of the effective Lagrangian density is BRST-exact, so these can also be expressed

in terms of BRST variation of gauge-fixing fermion ( $\Psi$ ). Thus, the effective Lagrangian density can also be expressed as

$$\mathcal{L} = \mathcal{L}_0 + \text{Tr}(s_b \Psi), \quad (2.10)$$

where the explicit form of the gauge-fixing fermion  $\Psi$  is

$$\Psi = -i\tilde{C} \left( \partial^\mu A_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu A_\mu \right). \quad (2.11)$$

One could also check that the effective Lagrangian density (2.7) is also invariant under the another nilpotent transformation where the role of ghost and anti-ghost fields are interchanged. These so-called anti-BRST transformations are

$$\begin{aligned} s_{ab} A_\mu &= \partial_\mu \tilde{C} - \frac{m^2}{n \cdot \partial} n_\mu \tilde{C} - i[A_\mu, \tilde{C}] + i \frac{m^2}{2} \left[ \tilde{C}, \frac{n \cdot A}{(n \cdot \partial)^2} \right] + i \frac{m^2}{2} n_\mu \left( \frac{1}{(n \cdot \partial)^2} n \cdot [A, \tilde{C}] \right), \\ &= \mathcal{D}_\mu \tilde{C}, \\ s_{ab} \tilde{C} &= i\tilde{C}^2, \quad s_{ab} C = -iB + iC\tilde{C}, \quad s_{ab} B = -i[B, \tilde{C}]. \end{aligned} \quad (2.12)$$

The gauge-fixing and ghost parts of the Lagrangian density are anit-BRST exact also and can also be described in terms of anti-BRST variation of some another gauge-fixing fermion. Here, we would like to state that the VSR gauge fields are massive with a common mass, however, in nature gauge fields may have different masses due to the spontaneous symmetry breaking in VSR with non-Abelian gauge symmetry Ref. [18]. In such a way, the fields can have the usual mass due to spontaneous symmetry breaking in addition to a flavor-dependent VSR mass.

## 2.1 VSR modified extended BRST invariant BV action

Now, there exists an interesting question that if the gauge field in VSR is being displaced as  $A_\mu \rightarrow A_\mu - \bar{A}_\mu$ : does the gauge symmetry still remain and moreover, how does this shift symmetry affect the underlying BRST structure. In this context, we shift all the fields (within VSR framework) from their original value as follows

$$A_\mu \longrightarrow A_\mu - \bar{A}_\mu, \quad C \longrightarrow C - \bar{C}, \quad \tilde{C} \longrightarrow \tilde{C} - \bar{\tilde{C}}, \quad B \longrightarrow B - \bar{B}. \quad (2.13)$$

Under these shifts, the Lagrangian density (2.7) in VSR is modified by

$$\bar{\mathcal{L}} = \mathcal{L}(A_\mu - \bar{A}_\mu, C - \bar{C}, \tilde{C} - \bar{\tilde{C}}, B - \bar{B}). \quad (2.14)$$

This shifted version of Lagrangian density remains invariant under the BRST transformation (2.9) with respect to the shifted fields  $A_\mu - \bar{A}_\mu, C - \bar{C}, \tilde{C} - \bar{\tilde{C}}, B - \bar{B}$ . In addition, this Lagrangian density is also invariant under the (local) shift symmetry  $s_b \phi = \mathcal{R}(x)$ ,  $s_b \bar{\phi} = \mathcal{R}(x)$ , where collective fields  $\phi$  and  $\bar{\phi}$  are  $(A_\mu, C, \tilde{C}, B)$  and  $(\bar{A}_\mu, \bar{C}, \bar{\tilde{C}}, \bar{B})$ , respectively.  $\mathcal{R}(x)$  is the generic notation for the Slavnov variations of fields  $\phi$  and  $\bar{\phi}$ . This deserves further being gauge fixed and, in turn, leads to an additional BRST symmetry. The BRST symmetry together with the shift symmetry is known as the extended BRST symmetry. This

extended BRST symmetry transformations corresponding to Lagrangian density (2.14) in VSR read

$$\begin{aligned} s_b A_\mu &= \psi_\mu, \quad s_b \bar{A}_\mu = \psi_\mu - \mathcal{D}_\mu^{(A-\bar{A})}(C - \bar{C}), \\ s_b C &= \epsilon, \quad s_b \bar{C} = \epsilon - i(C - \bar{C})^2, \quad s_b \tilde{C} = \tilde{\epsilon}, \\ s_b \tilde{\bar{C}} &= \tilde{\epsilon} - i(B - \bar{B}), \quad s_b B = \rho, \quad s_b \bar{B} = \rho, \end{aligned} \quad (2.15)$$

where

$$\begin{aligned} \mathcal{D}_\mu^{(A-\bar{A})}(C - \bar{C}) &= \partial_\mu(C - \bar{C}) - \frac{m^2}{n \cdot \partial} n_\mu(C - \bar{C}) \\ &\quad - i[A - \bar{A}, C - \bar{C}] + i \frac{m^2}{2} \left[ C - \bar{C}, \frac{n \cdot (A - \bar{A})}{(n \cdot \partial)^2} \right] \\ &\quad + i \frac{m^2}{2} n_\mu \left( \frac{1}{(n \cdot \partial)^2} n \cdot [A - \bar{A}, C - \bar{C}] \right). \end{aligned} \quad (2.16)$$

Here, extra fields  $\psi_\mu, \epsilon, \tilde{\epsilon}$  and  $\rho$  denote the ghost fields related to shift symmetry for  $A_\mu, C, \tilde{C}$  and  $B$ , respectively. Due to nilpotency property of extended BRST symmetry (2.15), it is evident that the variation of these ghost fields  $\psi_\mu, \epsilon, \tilde{\epsilon}$  and  $\rho$  under extended BRST transformation vanishes. Now, in order to make theory unchanged, we need to remove the contribution of these ghosts from the physical states. Thus, we introduce the antifields (anti-ghosts)  $A_\mu^*, C^*, \tilde{C}^*$  and  $B^*$  corresponding to each ghost field which compensates the net contribution of these ghosts. The BRST variation of these antifields are defined by

$$s_b A_\mu^* = -\zeta_\mu, \quad s_b C^* = -\sigma, \quad s_b \tilde{C}^* = -\tilde{\sigma}, \quad s_b B^* = -\tilde{v}, \quad (2.17)$$

where  $\zeta_\mu, \sigma, \tilde{\sigma}$  and  $\tilde{v}$  are auxiliary fields corresponding to shifted fields  $\bar{A}_\mu, \bar{C}, \tilde{\bar{C}}$  and  $\bar{B}$  and do not change under BRST transformation.

In order to fix the gauge for shift symmetry, we add following gauge fixing term to the VSR quantum action (2.7) (and we call the resulting Lagrangian density as BV action):

$$\begin{aligned} \bar{\mathcal{L}}_{gf} &= \text{Tr} \left[ -\zeta^\mu \bar{A}_\mu - A^{\mu*} \left[ \psi_\mu - \mathcal{D}_\mu^{(A-\bar{A})}(C - \bar{C}) \right] + \sigma \tilde{\bar{C}} - C^* [\tilde{\epsilon} - i(B - \bar{B})] \right. \\ &\quad \left. - \tilde{\sigma} \bar{C} + \tilde{C}^* [\epsilon - i(C - \bar{C})^2] + \tilde{v} \bar{B} + B^* \rho \right]. \end{aligned} \quad (2.18)$$

In this way, all the tilde fields will vanish and we then recover our original theory. We note that this gauge-fixed Lagrangian density,  $\bar{\mathcal{L}}_{gf}$ , is also invariant under the extended BRST symmetry transformations (2.15).

Now, by integrating out the auxiliary fields  $\zeta_\mu, \sigma, \tilde{\sigma}$  and  $\tilde{v}$ , this reads

$$\bar{\mathcal{L}}_{gf} = \text{Tr} \left[ -A_\mu^* (\psi^\mu - \mathcal{D}^\mu C) - C^* (\tilde{\epsilon}^a - iB) + \tilde{C}^* (\epsilon - iC^2) + B^* \rho \right]. \quad (2.19)$$

Since the gauge-fixed Lagrangian density (2.6) is BRST-exact, therefore one can express in terms of a general gauge-fixing fermion  $\Psi(A_\mu, \tilde{C}, C, B)$  as

$$\mathcal{L}_{gf} = s_b \Psi = -\frac{\delta \Psi}{\delta A_\mu} \psi_\mu + \frac{\delta \Psi}{\delta C} \epsilon + \frac{\delta \Psi}{\delta \tilde{C}} \tilde{\epsilon} - \frac{\delta \Psi}{\delta B} \rho, \quad (2.20)$$



In the last term, the BRST transformations (2.15) are utilized. Now, we utilize the equations of motion for the auxiliary fields which set all the fluctuated fields to zero. Thus, we left with the following BV action:

$$\begin{aligned}\mathcal{L}_{eff} &= \mathcal{L}_0 + \mathcal{L}_{gf} + \bar{\mathcal{L}}_{gf}, \\ &= \mathcal{L} + \text{Tr} \left[ \left( -A_\mu^* - \frac{\delta\Psi}{\delta A^\mu} \right) \psi^\mu + \left( \tilde{C}^* + \frac{\delta\Psi}{\delta C} \right) \epsilon - \left( C^* - \frac{\delta\Psi}{\delta \tilde{C}} \right) \tilde{\epsilon} \right. \\ &\quad \left. + \left( B^* - \frac{\delta\Psi}{\delta B} \right) \rho + A_\mu^* \mathcal{D}_\mu C + iC^* B - i\tilde{C}^* C^2 \right].\end{aligned}\quad (2.21)$$

In order to get identifications of the antifields in VSR, it is sufficient to integrate out the ghost fields associated with the shift symmetry

$$A_\mu^* = -\frac{\delta\Psi}{\delta A^\mu}, \quad \tilde{C}^* = -\frac{\delta\Psi}{\delta C}, \quad C^* = \frac{\delta\Psi}{\delta \tilde{C}}, \quad B^* = \frac{\delta\Psi}{\delta B}.\quad (2.22)$$

However, for the VSR modified gauge-fixing fermion given in (2.11), we determine the anti-ghost fields as following:

$$\begin{aligned}\bar{A}_\mu^{a*} &= -i\partial_\mu \tilde{C} + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{C}, \quad \tilde{C}^{a*} = 0, \quad \bar{C}^{a*} = -i\partial_\mu A^{a\mu} + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu A^{a\mu}, \\ B^{a*} &= 0.\end{aligned}\quad (2.23)$$

Here, we observe that, analogous to Lorentz invariant case, antifields get identification naturally. This clarifies the geometric interpretation of the antifields on the line of Maurer-Cartan 1-forms. Plugging these anti-ghost fields in (2.21), we can recover the original Lagrangian density of YM theory in VSR.

## 2.2 VSR modified extended BRST invariant superspace formulation

We know that superspace formulations for gauge theories can be built up in such a manner that the BRST transformations are realized as translations along the Grassmannian coordinate [46]. In order to describe the VSR modified BRST invariant BV action in superspace, we need an extra (Grassmannian) coordinate  $\theta$  together with  $x^\mu$ . Superspace formulations for the VSR modified BRST transformation are obtained by associating with each field a superfield of the form

$$\begin{aligned}\mathfrak{A}_\mu(x, \theta) &= A_\mu(x) + \theta \mathcal{D}_\mu C, \\ \mathfrak{C}(x, \theta) &= C(x) + i\theta C^2, \\ \tilde{\mathfrak{C}}(x, \theta) &= \tilde{C}(x) + i\theta B.\end{aligned}\quad (2.24)$$

The shifted superfields in VSR will be consistent only if these can be written by

$$\begin{aligned}\mathbb{A}_\mu(x, \theta) &= \mathfrak{A}_\mu(x, \theta) - \bar{\mathfrak{A}}_\mu(x, \theta) = (A_\mu - \bar{A}_\mu) + \theta \mathcal{D}_\mu^{(A-\bar{A})} (C - \bar{C}), \\ \mathbb{C}(x, \theta) &= \mathfrak{C}(x, \theta) - \bar{\mathfrak{C}}(x, \theta) = (C - \bar{C}) + i\theta (C - \bar{C})^2, \\ \tilde{\mathbb{C}}(x, \theta) &= \tilde{\mathfrak{C}}(x, \theta) - \bar{\tilde{\mathfrak{C}}}(x, \theta) = \tilde{C}(x) - \bar{\tilde{C}}(x) + i\theta (B - \bar{B}).\end{aligned}\quad (2.25)$$

From the above one can see the arbitrariness in the extended BRST symmetries. Therefore, one can not determine the individual superfields uniquely. So, to be consistent with the above analysis, we can define the original superfields and shift superfields with the help of extended BRST transformation as follows,

$$\begin{aligned}
\mathfrak{A}_\mu(x, \theta) &= A_\mu + \theta\psi_\mu, & \bar{\mathfrak{A}}_\mu(x, \theta) &= \bar{A}_\mu + \theta(\psi_\mu - \mathcal{D}_\mu^{(A-\bar{A})}(C - \bar{C})), \\
\mathfrak{C}(x, \theta) &= C + \theta\epsilon, & \bar{\mathfrak{C}}(x, \theta) &= \bar{C} + \theta(\epsilon - i(C - \bar{C})^2), \\
\tilde{\mathfrak{C}}(x, \theta) &= \tilde{C} + \theta\tilde{\epsilon}, & \bar{\tilde{\mathfrak{C}}}(x, \theta) &= \bar{\tilde{C}} - \theta(\tilde{\epsilon} - iB + i\tilde{B}), \\
\mathfrak{B}(x, \theta) &= B + \theta\rho.
\end{aligned} \tag{2.26}$$

Exploiting BRST transformations (2.17), we introduce the super antifields with one grassmannian coordinate in VSR as

$$\begin{aligned}
\bar{\mathfrak{A}}_\mu^*(x, \theta) &= A_\mu^* - \theta\zeta_\mu, \\
\bar{\mathfrak{C}}^*(x, \theta) &= C^* - \theta\sigma, \\
\bar{\tilde{\mathfrak{C}}}^*(x, \theta) &= \tilde{C}^* - \theta\tilde{\sigma}, \\
\mathfrak{B}^*(x, \theta) &= B^* - \theta\tilde{v}.
\end{aligned} \tag{2.27}$$

We find that the appropriate combinations of superfields of (2.26) and (2.27), leads to the gauge-fixed Lagrangian density corresponding to shift symmetry in VSR (2.18) as following:

$$\bar{\mathcal{L}}_{gf} = \text{Tr} \left[ \frac{\partial}{\partial\theta} \left( \bar{\mathfrak{A}}_\mu^* \mathfrak{A}^\mu + \bar{\mathfrak{C}}^* \bar{\mathfrak{C}} - \bar{\tilde{\mathfrak{C}}}^* \tilde{\mathfrak{C}} - \mathfrak{B}^* \mathfrak{B} \right) \right]. \tag{2.28}$$

The VSR modified gauge-fixed fermion (2.11) in extended BRST superspace formulation can be written as

$$\begin{aligned}
\Omega(x, \theta) &= -i\tilde{C} \left( \partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) A^\mu + i\theta \left[ \bar{C} \left( \partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) \psi^{\mu a} \right. \\
&\quad \left. - \tilde{\epsilon} \left( \partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) A^\mu \right].
\end{aligned} \tag{2.29}$$

Here, it is evident that  $\theta$  component of the above expression gives the gauge fixing Lagrangian density corresponding to the original BRST symmetry (2.6), i.e.,

$$\mathcal{L}_{gf} = \text{Tr} \left[ \frac{\partial}{\partial\theta} \Omega(x, \theta) \right], \tag{2.30}$$

Being the  $\theta$  component of a super gauge-fixed fermion, it is obvious that  $\mathcal{L}_{gf}$  is invariant under the extended BRST transformations.

### 2.3 VSR modified extended anti-BRST symmetry

In this subsection, we construct the VSR-modified extended anti-BRST transformation. The importance of anti-BRST transformation lies in the fact that, while the anti-BRST invariance does not lead to any additional information in comparison to BRST invariance,

it is extremely important in order to put the theory in geometrical setting. The extended anti-BRST transformations which leaves the BV action invariant are,

$$\begin{aligned}
s_{ab}A_\mu &= A_\mu^\star + \mathcal{D}_\mu^{(A-\bar{A})}(\tilde{C} - \bar{\tilde{C}}), \quad s_{ab}\bar{A}_\mu = A_\mu^\star, \\
s_{ab}\tilde{C} &= \tilde{C}^\star + i(\tilde{C} - \bar{\tilde{C}})^2, \quad s_{ab}\bar{\tilde{C}} = \tilde{C}^\star, \\
s_{ab}C &= C^\star - iB + i\bar{B} + i(C - \bar{C})(\tilde{C} - \bar{\tilde{C}}), \quad s_{ab}\bar{C} = C^\star, \\
s_{ab}B &= B^\star - i[(B - \bar{B}), \tilde{C} - \bar{\tilde{C}}], \quad s_{ab}\bar{B} = B^\star, \\
s_{ab}\psi_\mu &= \zeta_\mu + \mathcal{D}_\mu^{(A-\bar{A})}(B - \bar{B}) - [\mathcal{D}_\mu^{(A-\bar{A})}(C - \bar{C})](\tilde{C} - \bar{\tilde{C}}), \\
s_{ab}\epsilon &= \sigma - [B - \bar{B}, C - \bar{C}] + (\tilde{C} - \bar{\tilde{C}})(C - \bar{C})^2, \\
s_{ab}\tilde{\epsilon} &= \tilde{\sigma} - [B - \bar{B}, \tilde{C} - \bar{\tilde{C}}], \quad s_{ab}\rho = \tilde{v}.
\end{aligned} \tag{2.31}$$

Rest fields, whose anti-BRST transformations are not written here do not change under the extended anti-BRST transformation. To describe the superspace formulation of Yang-Mills theory in VSR having both the extended BRST and extended anti-BRST invariance, we need two additional Grassmannian coordinates  $\theta, \bar{\theta}$ . Now, it is straightforward to write the superfields in this formulation where the BRST and anti-BRST transformations merely correspond to translations in the  $\theta$  and  $\bar{\theta}$  coordinates respectively. Thus, we see that the results of superspace description of Lorentz invariant 1-form theory [42] also hold in the case of Lorentz breaking theory.

### 3 2-form gauge theory: VSR modified BV action in superspace

The study of Abelian 2-form gauge theory is important because it plays a crucial role in studying the theory for classical strings [25], vortex motion in an irrotational, incompressible fluid [26, 27] and the dual formulation of the Abelian Higgs model [28]. In this section, we discuss the VSR modified extended BRST and extended anti-BRST transformations (which include a shift symmetry) for the BV action of 2-form gauge theory. We further demonstrate a superspace description for the 2-form gauge theory having extended BRST invariance and extended anti-BRST invariance. To do so, we start with the classical Lagrangian density for Abelian rank-2 antisymmetric tensor field  $(B_{\mu\nu})$  theory in VSR as [7, 8]

$$\mathcal{L}_0 = \frac{1}{12}\tilde{F}_{\mu\nu\rho}\tilde{F}^{\mu\nu\rho}, \tag{3.1}$$

where  $\tilde{F}_{\mu\nu\rho}$  is the VSR-modified field-strength tensor defined as  $\tilde{F}_{\mu\nu\rho} \equiv \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} - \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\mu B_{\nu\rho} - \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\nu B_{\rho\mu} - \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\rho B_{\mu\nu}$ . Here  $n_\mu$  is a fixed null vector and transforms multiplicatively, as before, under a VSR transformation to ensure the invariance of non-local terms.

This field-strength tensor and, consequently, Lagrangian density is not invariant under the Lorentz invariant gauge transformation  $\delta B_{\mu\nu} = \partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu$ , where  $\zeta_\mu(x)$  is an arbitrary vector field. Rather, this is invariant under the following VSR-modified gauge transformation

$$\delta B_{\mu\nu} = \partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu - \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\mu \zeta_\nu + \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\nu \zeta_\mu. \tag{3.2}$$

Since the Lagrangian density is invariant under above non-local transformation, hence, to quantize this theory following BRST technique, it is necessary to introduce two anticommuting vector fields  $\rho_\mu$  and  $\tilde{\rho}_\mu$ , a commuting vector field  $\beta_\mu$ , two anticommuting scalar fields  $\chi$  and  $\tilde{\chi}$ , and the commuting scalar fields  $\sigma, \varphi$  and  $\tilde{\sigma}$  [31]. Involving all these fields, the gauge breaking term together with the ghosts is given as [7]

$$\begin{aligned}\mathcal{L}_{gf} = & i\tilde{\rho}_\nu \left( \partial_\mu \partial^\mu \rho^\nu - \partial_\mu \partial^\nu \rho^\mu - m^2 \rho^\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \partial \cdot \rho + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\nu n \cdot \rho - \frac{1}{4} \frac{m^4}{(n \cdot \partial)^2} n^\nu n \cdot \rho \right) \\ & - \tilde{\sigma} (\partial_\mu \partial^\mu - m^2) \sigma + \beta_\nu \partial_\mu B^{\mu\nu} - \frac{1}{2} m^2 \beta_\nu \frac{1}{n \cdot \partial} n_\mu B^{\mu\nu} + \lambda_1 \beta_\nu \beta^\nu \\ & - \beta_\nu \partial^\nu \varphi - i\tilde{\chi} \partial_\mu \rho^\mu + \frac{i}{2} m^2 \tilde{\chi} \frac{1}{n \cdot \partial} n_\mu \rho^\mu - i\lambda_2 \tilde{\chi} \chi - i\tilde{\rho}^\mu \partial_\mu \chi - \frac{i}{2} \frac{m^2}{n \cdot \partial} \tilde{\rho}^\mu n_\mu \chi,\end{aligned}\quad (3.3)$$

$k_1$  and  $k_2$  are arbitrary gauge parameters. The ghost and ghost of ghost propagators in momentum space are given, respectively, by [8]

$$\begin{aligned}D_{\mu\nu}^{gh}(k) &= -\frac{1}{k^2 + m^2} \left[ g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2} \right], \\ D^{ggh}(p) &= -\frac{1}{p^2 + m^2}.\end{aligned}\quad (3.4)$$

These expressions suggest that ghost and ghost of ghost have same mass  $m$ . Also, these propagators follow a large momentum behavior similar to the Lorentz-invariant case. Therefore, the 2-form theory in VSR is a renormalizable theory.

By incorporating the gauge breaking term (3.3), the Lagrangian density in VSR reads effectively

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{gf}, \quad (3.5)$$

which is invariant under the following nilpotent BRST transformation:

$$\begin{aligned}s_b B_{\mu\nu} &= (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \rho_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \rho_\mu), \\ s_b \rho_\mu &= -i\partial_\mu \sigma + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \sigma, \quad s_b \sigma = 0, \quad s_b \tilde{\rho}_\mu = i\beta_\mu, \\ s_b \beta_\mu &= 0, \quad s_b \tilde{\sigma} = -\tilde{\chi}, \quad s_b \tilde{\chi} = 0, \quad s_b \varphi = \chi, \quad s_b \chi = 0.\end{aligned}\quad (3.6)$$

Since the gauge-fixing and ghost part of the effective Lagrangian density is BRST-exact and therefore can be expressed in terms of BRST variation of some gauge-fixed fermion  $\Psi$ . Therefore,

$$\mathcal{L}_{gf} = s_b \Psi, \quad (3.7)$$

where  $\Psi$  has the following form:

$$\begin{aligned}\Psi = & -i \left[ \tilde{\rho}_\nu \partial_\mu B^{\mu\nu} + \tilde{\sigma} \partial_\mu \rho^\mu + \varphi \partial_\mu \tilde{\rho}^\mu - \tilde{\rho}_\nu k_1 \beta^\nu - \varphi k_2 \tilde{\chi} - \frac{\tilde{\rho}_\nu}{2} \frac{m^2}{n \cdot \partial} n_\mu B^{\mu\nu} \right. \\ & \left. - \frac{\tilde{\sigma}}{2} \frac{m^2}{n \cdot \partial} n_\mu \rho^\mu - \frac{\varphi}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}^\mu \right].\end{aligned}\quad (3.8)$$

This gauge-fixed fermion is very important to identify the antifields of BV action.

### 3.1 VSR modified extended BRST invariant BV action

The extended BRST and extended anti-BRST invariant formulations of the Lorentz invariant BV action lead to the proper identification of the antifields through equations of motion of auxiliary field variables [41, 42, 44]. The study of extended (including shift) symmetries introduced through collective fields is important because it ensure that Schwinger-Dyson equations at the level of the BRST algebra can be performed within the Feynman path integral [38]. In this subsection, we study the VSR modified extended BRST invariant BV action. To do so, we first deviate all the fields from their original values. This enlarges in a trivial way the symmetry content of the theory, adding extra shift symmetries. To study the extended BRST structure of the Abelian rank-2 tensor field theory in VSR, we shift all the fields of theory from their original values as follows  $B_{\mu\nu} - \bar{B}_{\mu\nu}, \rho_\mu - \bar{\rho}_\mu, \tilde{\rho}_\mu - \bar{\tilde{\rho}}_\mu, \sigma_\mu - \bar{\sigma}_\mu, \tilde{\sigma}_\mu - \bar{\tilde{\sigma}}_\mu, \beta_\mu - \bar{\beta}_\mu, \chi - \bar{\chi}, \tilde{\chi} - \bar{\tilde{\chi}}, \varphi - \bar{\varphi}$ . This leads to the following shifted Lagrangian density:

$$\bar{\mathcal{L}} = \mathcal{L}_0(B_{\mu\nu} - \bar{B}_{\mu\nu}) + \mathcal{L}_{gf}(B_{\mu\nu} - \bar{B}_{\mu\nu}, \Xi - \bar{\Xi}), \quad (3.9)$$

where  $\Xi - \bar{\Xi} = \rho_\mu - \bar{\rho}_\mu, \tilde{\rho}_\mu - \bar{\tilde{\rho}}_\mu, \sigma_\mu - \bar{\sigma}_\mu, \tilde{\sigma}_\mu - \bar{\tilde{\sigma}}_\mu, \beta_\mu - \bar{\beta}_\mu, \chi - \bar{\chi}, \tilde{\chi} - \bar{\tilde{\chi}}, \varphi - \bar{\varphi}$ .

The explicit form of  $\mathcal{L}_{gf}(B_{\mu\nu} - \bar{B}_{\mu\nu}, \Xi - \bar{\Xi})$  is given by

$$\begin{aligned} \mathcal{L}_{gf} = & -i \left[ \partial_\mu \tilde{\rho}_\nu \partial^\mu \rho^\nu + m^2 \tilde{\rho}_\nu \rho^\nu - \partial_\mu \tilde{\rho}_\nu \partial^\mu \bar{\rho}^\nu - m^2 \tilde{\rho}_\nu \bar{\rho}^\nu - \partial_\mu \tilde{\rho}_\nu \partial^\nu \rho^\mu + \frac{1}{2} \partial_\mu \tilde{\rho}_\nu \frac{m^2}{n \cdot \partial} n^\nu \rho^\mu \right. \\ & + \partial^\nu \rho^\mu \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}_\nu - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_\mu \tilde{\rho}_\nu n^\nu \rho^\mu + \partial_\mu \tilde{\rho}_\nu \partial^\nu \bar{\rho}^\mu - \frac{1}{2} \partial_\mu \tilde{\rho}_\nu \frac{m^2}{n \cdot \partial} n^\nu \bar{\rho}^\mu \\ & - \frac{1}{2} \partial^\nu \bar{\rho}^\mu \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}_\nu + \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_\mu \tilde{\rho}_\nu n^\nu \bar{\rho}^\mu - \partial_\mu \tilde{\rho}_\nu \partial^\mu \rho^\nu - m^2 \tilde{\rho}_\nu \rho^\nu + \partial_\mu \tilde{\rho}_\nu \partial^\mu \bar{\rho}^\nu \\ & + m^2 \tilde{\rho}_\nu \bar{\rho}^\nu + \partial_\mu \tilde{\rho}_\nu \partial^\nu \rho^\mu - \frac{1}{2} \partial^\mu \tilde{\rho}^\mu \frac{m^2}{n \cdot \partial} n^\nu \bar{\rho}^\mu - \frac{1}{2} \partial^\nu \rho^\mu \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}^\nu + \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_\mu \tilde{\rho}_\nu n^\nu \rho^\mu \\ & - \partial_\mu \tilde{\rho}_\nu \partial^\nu \bar{\rho}^\mu + \frac{1}{2} \partial_\mu \tilde{\rho}_\nu \frac{m^2}{n \cdot \partial} n^\nu \bar{\rho}^\mu + \frac{1}{2} \partial^\nu \bar{\rho}^\mu \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}^\nu - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_\mu \tilde{\rho}_\nu n^\nu \bar{\rho}^\mu + \partial_\mu \tilde{\sigma} \partial^\mu \sigma \\ & - \partial_\mu \tilde{\sigma} \partial^\mu \bar{\sigma} - \partial_\mu \tilde{\sigma} \partial^\mu \sigma + \partial_\mu \tilde{\sigma} \partial^\mu \bar{\sigma} + m^2 \tilde{\sigma} \sigma - m^2 \tilde{\sigma} \bar{\sigma} - m^2 \tilde{\sigma} \sigma + m^2 \tilde{\sigma} \bar{\sigma} \\ & + \beta_\nu \partial_\mu B^{\mu\nu} - \beta_\nu \partial_\mu \bar{B}^{\mu\nu} + \beta_\nu \partial^\nu \bar{\varphi} - \bar{\beta}_\nu \partial_\mu B^{\mu\nu} - \bar{\beta}_\nu \partial_\mu \bar{B}^{\mu\nu} + \bar{\beta}_\nu \partial^\nu \varphi + \bar{\beta} \partial^\nu \bar{\varphi} - \beta_\nu \partial^\nu \varphi \\ & - \frac{1}{2} \beta_\nu \frac{m^2}{n \cdot \partial} n_\mu B^{\mu\nu} + \frac{1}{2} \beta_\nu \frac{m^2}{n \cdot \partial} n_\mu \bar{B}^{\mu\nu} - \frac{\beta_\nu}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{\varphi} + \frac{\bar{\beta}_\nu}{2} \frac{m^2}{n \cdot \partial} n_\mu B^{\mu\nu} + \frac{\bar{\beta}_\nu}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}^{\mu\nu} \\ & - \frac{\beta_\nu}{2} \frac{m^2}{n \cdot \partial} n^\nu \varphi - \frac{\bar{\beta}_\nu}{2} \frac{m^2}{n \cdot \partial} n^\nu \varphi + \frac{\bar{\beta}_\nu}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{\varphi} + k_1 \beta_\nu \beta^\nu - k_1 \beta_\nu \bar{\beta}^\nu - k_1 \bar{\beta}_\nu \beta^\nu + k_1 \bar{\beta}_\nu \bar{\beta}^\nu \\ & - i \left( \tilde{\chi} \partial_\mu \rho^\mu - \tilde{\chi} \partial_\mu \bar{\rho}^\mu - \bar{\tilde{\chi}} \partial_\mu \rho^\mu + \bar{\tilde{\chi}} \partial_\mu \bar{\rho}^\mu - \frac{\bar{\chi}}{2} \frac{m^2}{n \cdot \partial} n \cdot \rho + \frac{\bar{\chi}}{2} \frac{m^2}{n \cdot \partial} n \cdot \bar{\rho} + \frac{\bar{\chi}}{2} \frac{m^2}{n \cdot \partial} n \cdot \rho \right. \\ & - \frac{\bar{\chi}}{2} \frac{m^2}{n \cdot \partial} n \cdot \bar{\rho} \left. \right) - i \left( \chi \partial_\mu \tilde{\rho}^\mu - \chi \partial_\mu \bar{\tilde{\rho}}^\mu - \bar{\chi} \partial_\mu \tilde{\rho}^\mu + \bar{\chi} \partial_\mu \bar{\tilde{\rho}}^\mu - \frac{\chi}{2} \frac{m^2}{n \cdot \partial} n \cdot \tilde{\rho} + \frac{\chi}{2} \frac{m^2}{n \cdot \partial} n \cdot \bar{\tilde{\rho}} \right. \\ & \left. + \frac{\bar{\chi}}{2} \frac{m^2}{n \cdot \partial} n \cdot \tilde{\rho} - \frac{\chi}{2} \frac{m^2}{n \cdot \partial} n \cdot \bar{\tilde{\rho}} \right) - k_2 \chi \tilde{\chi} + k_2 \chi \bar{\tilde{\chi}} + k_2 \bar{\chi} \tilde{\chi} - k_2 \bar{\chi} \bar{\tilde{\chi}} \left. \right]. \quad (3.10) \end{aligned}$$

This Lagrangian density coincides with  $\mathcal{L}_{gf}$  of (3.3) when bar fields vanish. It is evident that this Lagrangian density is invariant under the BRST transformation (3.6) for the

shifted fields. In addition, there exists the following shift symmetry also:

$$s_b \Phi(x) = \alpha(x), \quad s_b \bar{\Phi}(x) = \alpha(x), \quad (3.11)$$

which leaves this Lagrangian density invariant. Here  $\Phi$  and  $\bar{\Phi}$  are generic notation for all fields and shifted fields respectively. The form the extended BRST symmetry. The extended BRST transformation, which is comprised by the BRST symmetry along with the above shift symmetry, is then given by

$$s_b \Phi(x) = \alpha(x), \quad s_b \bar{\Phi}(x) = \alpha(x) - \beta(x), \quad (3.12)$$

where  $\beta(x)$  refers the original BRST transformation collectively, whereas  $\alpha(x)$  refers the shift transformation collectively. In order to quantize theory collectively, we need to fix the gauge for all the local symmetry. Therefore, corresponding to this local shift symmetry, one needs the theory to be gauge-fixed and this leads to an additional BRST symmetry [42]. The extended BRST symmetry transformation for all the fields are given by

$$\begin{aligned} s_b B_{\mu\nu} &= \psi_{\mu\nu}, \quad s_b \bar{B}_{\mu\nu} = \psi_{\mu\nu} - (\partial_\mu \rho_\nu - \partial_\mu \bar{\rho}_\nu - \partial_\nu \rho_\mu + \partial_\nu \bar{\rho}_\mu \\ &\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \rho_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\rho}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\rho}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\rho}_\mu), \\ s_b \bar{\rho}_\mu &= \epsilon_\mu + i \partial_\mu \sigma - i \partial_\mu \bar{\sigma} + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \sigma + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\sigma}, \\ s_b \tilde{\rho}_\mu &= \xi_\mu, \quad s_b \bar{\tilde{\rho}}_\mu = \xi_\mu - i \beta_\mu + i \bar{\beta}_\mu, \quad s_b \bar{\sigma} = \varepsilon, \\ s_b \sigma &= \varepsilon, \quad s_b \beta_\mu = \eta_\mu, \quad s_b \bar{\beta}_\mu = \eta_\mu, \quad s_b \tilde{\sigma} = \psi, \quad s_b \tilde{\chi} = \eta, \\ s_b \bar{\tilde{\sigma}} &= \psi + \tilde{\chi} - \bar{\tilde{\chi}}, \quad s_b \bar{\tilde{\chi}} = \eta, \quad s_b \bar{\varphi} = \phi - \chi + \bar{\chi}, \quad s_b \chi = \Sigma, \\ s_b \varphi &= \phi, \quad s_b \bar{\chi} = \Sigma, \quad s_b \xi_i = 0, \quad \xi_i \equiv [\psi_{\mu\nu}, \epsilon_\mu, \xi_\mu, \varepsilon, \eta_\mu, \psi, \eta, \phi, \Sigma]. \end{aligned} \quad (3.14)$$

The fields  $\tilde{\psi}_{\mu\nu}$ ,  $\epsilon_\mu$ ,  $\xi_\mu$ ,  $\varepsilon$ ,  $\eta_\mu$ ,  $\psi$ ,  $\eta$ ,  $\phi$  and  $\Sigma$  are introduced as ghost fields associated with the shift symmetry corresponding to the fields  $B_{\mu\nu}$ ,  $\rho_\mu$ ,  $\tilde{\rho}_\mu$ ,  $\sigma$ ,  $\beta_\mu$ ,  $\tilde{\sigma}$ ,  $\tilde{\chi}$ ,  $\varphi$  and  $\chi$  respectively. Further, we add following antighost fields  $B_{\mu\nu}^*$ ,  $\rho_\mu^*$ ,  $\tilde{\rho}_\mu^*$ ,  $\sigma^*$ ,  $\tilde{\sigma}^*$ ,  $\beta_\mu^*$ ,  $\chi^*$ ,  $\tilde{\chi}^*$  and  $\varphi^*$  corresponding to the fields  $B_{\mu\nu}$ ,  $\rho_\mu$ ,  $\tilde{\rho}_\mu$ ,  $\sigma$ ,  $\beta_\mu$ ,  $\tilde{\sigma}$ ,  $\tilde{\chi}$ ,  $\varphi$  and  $\chi$  respectively with opposite statistics. These antighost fields transform under BRST transformations as following:

$$\begin{aligned} s_b B_{\mu\nu}^* &= L_{\mu\nu}, \quad s_b \rho_\mu^* = M_\mu, \quad s_b \tilde{\rho}_\mu^* = \bar{M}_\mu, \quad s_b \sigma^* = N, \\ s_b \tilde{\sigma}^* &= \bar{N}, \quad s_b \beta_\mu^* = S_\mu, \quad s_b \chi^* = O, \quad s_b \tilde{\chi}^* = \bar{O}, \quad s_b \varphi^* = T, \end{aligned} \quad (3.15)$$

where  $L_{\mu\nu}$ ,  $M_\mu$ ,  $\bar{M}_\mu$ ,  $N$ ,  $\bar{N}$ ,  $S_\mu$ ,  $O$ ,  $\bar{O}$ ,  $T$  are the Nakanishi-Lautrup type auxiliary fields and do not change under BRST transformation which ensure the nilpotency of BRST symmetry.

Now, we fix the gauge for shift symmetry in VSR by choosing the following gauge-fixed Lagrangian density:

$$\begin{aligned} \bar{\mathcal{L}}_{gf} &= L_{\mu\nu} \bar{B}^{\mu\nu} - B_{\mu\nu}^* (\psi^{\mu\nu} - \partial^\mu \rho^\nu + \partial^\mu \bar{\rho}^\nu + \partial^\nu \rho^\mu - \partial^\nu \bar{\rho}^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \rho^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{\rho}^\nu \\ &\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \rho^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{\rho}^\mu) + \bar{M}_\mu \bar{\rho}^\mu + \tilde{\rho}_\mu^* (\epsilon^\mu + i \partial^\mu \sigma - i \partial^\mu \bar{\sigma} - i \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \sigma \\ &\quad + i \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{\sigma}) + M_\mu \tilde{\rho}^\mu + \rho_\mu^* (\xi^\mu - i \beta^\mu + i \bar{\beta}^\mu) + N \bar{\sigma} - \sigma^* \varepsilon + \bar{N} \bar{\tilde{\sigma}} - \tilde{\sigma}^* (\psi - \tilde{\chi} + \bar{\tilde{\chi}}) \\ &\quad + \bar{O} \bar{\tilde{\chi}} + \tilde{\chi}^* \Sigma + O \bar{\tilde{\chi}} + \chi^* \eta + T \bar{\varphi} - \varphi^* (\phi - \chi + \bar{\chi}) + S_\mu \bar{\beta}^\mu - \beta_\mu^* \eta^\mu, \end{aligned} \quad (3.16)$$

which sets all the bar fields to zero and thus we recover the original theory. This gauge-fixing term is also invariant under the extended BRST symmetry transformations given in Eqs. (3.14) and (3.15). The gauge-fixed extended Lagrangian density,  $\tilde{\mathcal{L}}_{gf}$ , after exploiting the equations of motion of auxiliary fields has the following form:

$$\begin{aligned}\tilde{\mathcal{L}}_{gf} = & -B_{\mu\nu}^*(\psi^{\mu\nu} - \partial^\mu \rho^\nu + \partial^\nu \rho^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \rho^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \rho^\mu) + \tilde{\rho}_\mu^*(\epsilon^\mu + i\partial^\mu \sigma \\ & - \frac{i}{2} \frac{m^2}{n \cdot \partial} n^\mu \sigma) + \rho_\mu^*(\xi^\mu - i\beta^\mu) - \sigma^* \epsilon - \tilde{\sigma}^*(\psi - \tilde{\chi}) + \tilde{\chi}^* \Sigma + \chi^* \eta - \varphi^*(\phi - \chi) \\ & - \beta_\mu^* \eta^\mu.\end{aligned}\quad (3.17)$$

As the gauge-fixed fermion  $\Psi$  for Abelian rank-2 antisymmetric tensor field in VSR depends only on the original fields, then the most general gauge-fixed Lagrangian density is given by

$$\begin{aligned}\mathcal{L}_{gf} = s_b \Psi = & \psi_{\mu\nu} \frac{\delta \Psi}{\delta B_{\mu\nu}} + \epsilon_\mu \frac{\delta \Psi}{\delta \rho_\mu} + \xi_\mu \frac{\delta \Psi}{\delta \tilde{\rho}_\mu} + \varepsilon \frac{\delta \Psi}{\delta \sigma} \\ & + \psi \frac{\delta \Psi}{\delta \tilde{\sigma}} + \eta_\mu \frac{\delta \Psi}{\delta \beta_\mu} + \Sigma \frac{\delta \Psi}{\delta \chi} + \eta \frac{\delta \Psi}{\delta \tilde{\chi}} + \phi \frac{\delta \Psi}{\delta \varphi}.\end{aligned}\quad (3.18)$$

Utilizing the fermionic and bosonic behavior of fields, this can further be written as

$$\begin{aligned}\mathcal{L}_{gf} = & -\frac{\delta \Psi}{\delta B_{\mu\nu}} \psi_{\mu\nu} + \frac{\delta \Psi}{\delta \rho_\mu} \epsilon_\mu + \frac{\delta \Psi}{\delta \tilde{\rho}_\mu} \xi_\mu - \frac{\delta \Psi}{\delta \sigma} \varepsilon \\ & - \frac{\delta \Psi}{\delta \tilde{\sigma}} \psi - \frac{\delta \Psi}{\delta \beta_\mu} \eta_\mu + \frac{\delta \Psi}{\delta \chi} \Sigma + \frac{\delta \Psi}{\delta \tilde{\chi}} \eta - \frac{\delta \Psi}{\delta \varphi} \phi.\end{aligned}\quad (3.19)$$

Now, we are able to write the effective Lagrangian density in VSR,  $\mathcal{L}_{eff} = \mathcal{L}_0 + \mathcal{L}_{gf} + \tilde{\mathcal{L}}_{gf}$ , as follows

$$\begin{aligned}\mathcal{L}_{eff} = & \frac{1}{12} \tilde{F}_{\mu\nu\rho} \tilde{F}^{\mu\nu\rho} + B_{\mu\nu}^* \left( \partial^\mu \rho^\nu - \partial^\nu \rho^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \rho^\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \rho^\mu \right) \\ & + i\tilde{\rho}_\mu^* \left( \partial^\mu \sigma - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \sigma \right) - i\rho_\mu^* \beta^\mu + \tilde{\sigma}^* \tilde{\chi} + \varphi^* \chi - \left( B_{\mu\nu}^* + \frac{\delta \Psi}{\delta B_{\mu\nu}} \right) \psi^{\mu\nu} \\ & + \left( \rho_\mu^* + \frac{\delta \Psi}{\delta \tilde{\rho}^\mu} \right) \xi^\mu + \left( \tilde{\rho}_\mu^* + \frac{\delta \Psi}{\delta \rho^\mu} \right) \epsilon^\mu - \left( \sigma^* + \frac{\delta \Psi}{\delta \sigma} \right) \varepsilon - \left( \tilde{\sigma}^* + \frac{\delta \Psi}{\delta \tilde{\sigma}} \right) \psi \\ & + \left( \tilde{\chi}^* + \frac{\delta \Psi}{\delta \chi} \right) \Sigma + \left( \chi^* + \frac{\delta \Psi}{\delta \tilde{\chi}} \right) \eta - \left( \varphi^* + \frac{\delta \Psi}{\delta \varphi} \right) \phi - \left( \beta_\mu^* + \frac{\delta \Psi}{\delta \beta_\mu} \right) \eta^\mu,\end{aligned}\quad (3.20)$$

here expressions (3.1), (3.17) and (3.19) are utilized. Exploiting the equations of motion of ghost fields associated with the shift symmetry, we get the following identifications of antighost fields:

$$\begin{aligned}B_{\mu\nu}^* = & -\frac{\delta \Psi}{\delta B_{\mu\nu}}, \quad \tilde{\rho}_\mu^* = -\frac{\delta \Psi}{\delta \rho^\mu}, \quad \rho_\mu^* = -\frac{\delta \Psi}{\delta \tilde{\rho}^\mu}, \\ \sigma^* = & -\frac{\delta \Psi}{\delta \sigma}, \quad \tilde{\sigma}^* = -\frac{\delta \Psi}{\delta \tilde{\sigma}}, \quad \tilde{\chi}^* = -\frac{\delta \Psi}{\delta \chi}, \\ \chi^* = & -\frac{\delta \Psi}{\delta \tilde{\chi}}, \quad \beta_\mu^* = -\frac{\delta \Psi}{\delta \beta^\mu}, \quad \varphi^* = -\frac{\delta \Psi}{\delta \varphi}.\end{aligned}\quad (3.21)$$

With a particular expression of gauge-fixed fermion  $\Psi$  as given in (3.8), these leads to

$$B_{\mu\nu}^* = -i \left( \partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) \tilde{\rho}_\nu, \quad \tilde{\rho}_\mu^* = -i \left( \partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) \tilde{\sigma}, \quad (3.22)$$

$$\begin{aligned} \rho_\mu^* &= i \left( \partial^\nu B_{\nu\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu B_{\nu\mu} - k_1 \beta_\mu \right), \quad \sigma^* = 0, \\ \tilde{\sigma}^* &= i \left( \partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) \rho^\mu, \quad \tilde{\chi}^* = 0, \quad \chi^* = -ik_2 \varphi, \\ \beta_\mu^* &= ik_1 \tilde{\rho}_\mu, \quad \tilde{\varphi}^* = i \left( \partial_\mu \tilde{\rho}^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}^\mu - k_2 \tilde{\chi} \right). \end{aligned} \quad (3.23)$$

Here we see that these antighost fields coincide with the antifields of the theory. We note that these antifields are nonlocal which describe the features of VSR. Thus, analogous to Lorentz invariant theory, the antifields are obtained naturally in VSR also. One can see that these identifications lead the effective Lagrangian density to their original form as given in Eq. (3.5). Now, we can describe the gauge-fixing part of the effective Lagrangian density in terms of the BRST variation of a generalized gauge-fixed fermion as follows

$$\begin{aligned} \mathcal{L}_{eff} &= \mathcal{L}_0(B_{\mu\nu} - \bar{B}_{\mu\nu}) + s_b (B_{\mu\nu}^* \bar{B}^{\mu\nu} + \rho_\mu^* \tilde{\rho}^\mu + \tilde{\rho}_\mu^* \bar{\rho}^\mu + \sigma^* \bar{\sigma} + \tilde{\sigma}^* \bar{\tilde{\sigma}} + \beta_\mu^* \bar{\beta}^\mu \\ &\quad + \chi^* \bar{\tilde{\chi}} + \tilde{\chi}^* \bar{\chi} + \varphi^* \bar{\varphi}), \\ &\equiv \mathcal{L}_0(B_{\mu\nu} - \bar{B}_{\mu\nu}) + s_b \Phi^* \bar{\Phi}. \end{aligned} \quad (3.24)$$

Here  $\Phi^*$  and  $\bar{\Phi}$  are the generic notation for antifields and (corresponding) shifted fields, respectively. The ghost number of  $\Phi^* \bar{\Phi}$  is  $-1$ . We thus recover the BV action for Abelian 2-form gauge theory in VSR with the identification of antifields.

### 3.2 VSR modified extended BRST invariant superspace formulation

In this section we discuss a superspace formalism of the VSR modified 2-form theory having extended BRST invariance. In this regard, we extend the space to a superspace  $(x^\mu, \theta)$  by introducing a fermionic coordinate  $\theta$ . In this superspace, the “superconnection” 2-form is defined by

$$\omega^{(2)} = \frac{1}{2!} \mathcal{B}_{\mu\nu}(x, \theta) (dx^\mu \wedge dx^\nu) + \mathcal{M}_\mu(x, \theta) (dx^\mu \wedge d\theta) + \mathcal{N}(x, \theta) (d\theta \wedge d\theta), \quad (3.25)$$

where  $d = dx^\mu \left( \partial_\mu - \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) + d\theta \left( \partial_\theta - \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\theta \right)$  is the exterior derivative. By requiring the field strength,  $F^{(3)} = d\omega^{(2)}$ , to vanish along the  $\theta$  direction, we get the following form of the component of the superfields in VSR

$$\begin{aligned} \mathcal{B}_{\mu\nu}(x, \theta) &= B_{\mu\nu}(x) + \theta(s_b B_{\mu\nu}), \\ \mathcal{M}_\mu(x, \theta) &= \rho_\mu(x) + \theta(s_b \rho_\mu), \\ \mathcal{N}(x, \theta) &= \sigma(x) + \theta(s_b \sigma). \end{aligned} \quad (3.26)$$



In the similar fashion, we are able to write all the fields involved in extended BV action in superspace as

$$\begin{aligned}
\mathcal{B}_{\mu\nu}(x, \theta) &= B_{\mu\nu}(x) + \theta\psi_{\mu\nu}, \quad \mathcal{M}_\mu(x, \theta) = \rho_\mu(x) + \theta\epsilon_\mu, \\
\bar{\mathcal{B}}_{\mu\nu}(x, \theta) &= \bar{B}_{\mu\nu}(x) + \theta(\psi_{\mu\nu} - \partial_\mu\rho_\nu + \partial_\mu\bar{\rho}_\nu + \partial_\nu\rho_\mu - \partial_\nu\bar{\rho}_\mu + \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\mu\rho_\nu \\
&\quad - \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\mu\bar{\rho}_\nu - \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\nu\rho_\mu + \frac{1}{2}\frac{m^2}{n \cdot \partial}n_\nu\bar{\rho}_\mu), \\
\bar{\mathcal{M}}_\mu(x, \theta) &= \bar{\rho}_\mu(x) + \theta(\epsilon_\mu - i\partial_\mu\sigma + i\partial_\mu\bar{\sigma} + \frac{i}{2}\frac{m^2}{n \cdot \partial}n_\mu\sigma - \frac{i}{2}\frac{m^2}{n \cdot \partial}n_\mu\bar{\sigma}), \\
\mathcal{N}(x, \theta) &= \sigma(x) + \theta\varepsilon, \quad \bar{\mathcal{N}}(x, \theta) = \bar{\sigma}(x) + \theta\bar{\varepsilon}, \\
\tilde{\mathcal{M}}_\mu(x, \theta) &= \tilde{\rho}_\mu(x) + \theta\xi_\mu, \quad \bar{\tilde{\mathcal{M}}}_\mu(x, \theta) = \bar{\tilde{\rho}}_\mu(x) + \theta(\xi_\mu - i\beta_\mu + i\bar{\beta}_\mu), \\
\mathcal{S}_\mu(x, \theta) &= \beta_\mu(x) + \theta\eta_\mu, \quad \bar{\mathcal{S}}_\mu(x, \theta) = \bar{\beta}_\mu(x) + \theta\bar{\eta}_\mu, \\
\tilde{\mathcal{N}}(x, \theta) &= \tilde{\sigma}(x) + \theta\psi, \quad \bar{\tilde{\mathcal{N}}}(x, \theta) = \bar{\tilde{\sigma}}(x) + \theta(\psi - \tilde{\chi} + \tilde{\bar{\psi}}), \\
\mathcal{O}(x, \theta) &= \chi(x) + \theta\Sigma, \quad \bar{\mathcal{O}}(x, \theta) = \bar{\chi}(x) + \theta\bar{\Sigma}, \\
\tilde{\mathcal{O}}(x, \theta) &= \tilde{\chi}(x) + \theta\eta, \quad \bar{\tilde{\mathcal{O}}}(x, \theta) = \bar{\tilde{\chi}}(x) + \theta\bar{\eta}, \\
\mathcal{T}(x, \theta) &= \varphi(x) + \theta\phi, \quad \bar{\mathcal{T}}(x, \theta) = \bar{\varphi}(x) + \theta(\phi - \chi + \bar{\chi}).
\end{aligned} \tag{3.27}$$

The components of antifields of the theory in superspace is written by

$$\begin{aligned}
\bar{\mathcal{B}}_{\mu\nu}^* &= B_{\mu\nu}^* + \theta L_{\mu\nu}, \quad \bar{\mathcal{M}}_\mu^* = \rho_\mu^* + \theta M_\mu, \quad \bar{\tilde{\mathcal{M}}}_\mu^* = \tilde{\rho}_\mu^* + \theta \tilde{M}_\mu, \\
\bar{\mathcal{S}}_\mu^* &= \beta_\mu^* + \theta S_\mu, \quad \bar{\mathcal{N}}^* = \sigma^* + \theta N, \quad \bar{\tilde{\mathcal{N}}}^* = \tilde{\sigma}^* + \theta \tilde{N}, \\
\bar{\mathcal{O}}^* &= \chi^* + \theta O, \quad \bar{\mathcal{T}}^* = \varphi^* + \theta T, \quad \bar{\tilde{\mathcal{O}}}^* = \tilde{\chi}^* + \theta \tilde{O}.
\end{aligned} \tag{3.28}$$

Exploiting expressions (3.27) and (3.28), we derive

$$\begin{aligned}
\frac{\delta}{\delta\theta}\bar{\mathcal{B}}_{\mu\nu}^*\bar{\mathcal{B}}^{\mu\nu} &= L_{\mu\nu}\bar{B}^{\mu\nu} - B_{\mu\nu}^*\left(\psi^{\mu\nu} - \partial^\mu\rho^\nu + \partial^\mu\bar{\rho}^\nu + \partial^\nu\rho^\mu - \partial^\nu\bar{\rho}^\mu + \frac{1}{2}\frac{m^2}{n \cdot \partial}n^\mu\rho^\nu \right. \\
&\quad \left. - \frac{1}{2}\frac{m^2}{n \cdot \partial}n^\mu\bar{\rho}^\nu - \frac{1}{2}\frac{m^2}{n \cdot \partial}n^\nu\rho^\mu + \frac{1}{2}\frac{m^2}{n \cdot \partial}n^\nu\bar{\rho}^\mu\right), \\
\frac{\delta}{\delta\theta}\bar{\tilde{\mathcal{M}}}_\mu^*\bar{\mathcal{M}}^\mu &= \bar{M}_\mu\bar{\rho}^\mu + \tilde{\rho}_\mu^*\left(\epsilon^\mu + i\partial^\mu\sigma - i\partial^\mu\bar{\sigma} - \frac{i}{2}\frac{m^2}{n \cdot \partial}n^\mu\sigma + \frac{i}{2}\frac{m^2}{n \cdot \partial}n^\mu\bar{\sigma}\right), \\
\frac{\delta}{\delta\theta}\bar{\tilde{\mathcal{M}}}_\mu\bar{\mathcal{M}}^{\mu*} &= M_\mu\bar{\rho}^\mu + \rho_\mu^*(\xi^\mu - i\beta^\mu + i\bar{\beta}^\mu), \\
\frac{\delta}{\delta\theta}\bar{\mathcal{N}}^*\bar{\mathcal{N}} &= N\bar{\sigma} - \sigma^*\varepsilon, \quad \frac{\delta}{\delta\theta}\bar{\tilde{\mathcal{N}}}^*\bar{\tilde{\mathcal{N}}} = \tilde{N}\bar{\tilde{\sigma}} - \tilde{\sigma}^*(\psi - \tilde{\chi} + \tilde{\bar{\psi}}), \\
\frac{\delta}{\delta\theta}\bar{\mathcal{O}}^*\bar{\mathcal{O}} &= \bar{O}\bar{\chi} + \tilde{\chi}^*\Sigma, \quad \frac{\delta}{\delta\theta}\bar{\tilde{\mathcal{O}}}^*\bar{\tilde{\mathcal{O}}} = \bar{\tilde{O}}\bar{\tilde{\chi}} + \chi^*\eta, \\
\frac{\delta}{\delta\theta}\bar{\mathcal{T}}^*\bar{\mathcal{T}} &= T\bar{\varphi} - \varphi^*(\phi - \chi + \bar{\chi}), \quad \frac{\delta}{\delta\theta}\bar{\mathcal{S}}_\mu^*\bar{\mathcal{S}}^\mu = S_\mu\bar{\beta}^\mu - \beta_\mu^*\eta^\mu.
\end{aligned} \tag{3.29}$$

Here, we note that the RHS of the sum of above expressions coincides with the gauge-fixed Lagrangian density corresponding to the shift symmetry (3.16). Thus, the gauge-fixed

Lagrangian density in superspace can be written as

$$\begin{aligned}\bar{\mathcal{L}}_{gf} = & \frac{\delta}{\delta\theta} \left[ \bar{\mathcal{B}}_{\mu\nu}^* \bar{\mathcal{B}}^{\mu\nu} + \bar{\mathcal{M}}_\mu^* \bar{\mathcal{M}}^\mu + \bar{\mathcal{M}}_\mu \bar{\mathcal{M}}^{\mu*} + \bar{\mathcal{N}}^* \bar{\mathcal{N}} + \bar{\mathcal{N}}^* \bar{\mathcal{N}} + \bar{\mathcal{O}}^* \bar{\mathcal{O}} + \bar{\mathcal{O}} \bar{\mathcal{O}}^* \right. \\ & \left. + \bar{\mathcal{T}}^* \bar{\mathcal{T}} + \bar{\mathcal{S}}_\mu^* \bar{\mathcal{S}}^\mu \right].\end{aligned}\quad (3.30)$$

Similar to the Lorentz invariant theory, the invariance of  $\bar{\mathcal{L}}_{gf}$  under the extended BRST transformation is evident from the above expression as it belongs to the  $\theta$  component of superfields. As the gauge-fixing fermion depends on the original fields, the component form of fermionic superfield  $\Gamma(x, \theta)$  in superspace is defined as

$$\begin{aligned}\Gamma(x, \theta) = & \Psi(x) + \theta \left[ -\frac{\delta\Psi}{\delta B_{\mu\nu}} \psi_{\mu\nu} + \frac{\delta\Psi}{\delta \rho_\mu} \epsilon_\mu + \frac{\delta\Psi}{\delta \tilde{\rho}_\mu} \xi_\mu - \frac{\delta\Psi}{\delta \sigma} \varepsilon - \frac{\delta\Psi}{\delta \tilde{\sigma}} \psi - \frac{\delta\Psi}{\delta \beta_\mu} \eta_\mu + \frac{\delta\Psi}{\delta \chi} \Sigma \right. \\ & \left. + \frac{\delta\Psi}{\delta \tilde{\chi}} \eta - \frac{\delta\Psi}{\delta \varphi} \phi \right].\end{aligned}\quad (3.31)$$

From the above expression, the most general VSR-modified gauge-fixed Lagrangian density  $\mathcal{L}_{gf}$  can be described in the superspace by

$$\mathcal{L}_{gf} = \frac{\delta\Gamma(x, \theta)}{\delta\theta}.\quad (3.32)$$

Being the  $\theta$  component of fermionic superfield, it is evident that  $\mathcal{L}_{gf}$  is invariant under the extended BRST transformation. Thus, the VSR-modified effective Lagrangian density in this superspace formalism is identified as

$$\begin{aligned}\mathcal{L}_{eff} = & \mathcal{L}_0(B_{\mu\nu} - \bar{B}_{\mu\nu}) + \frac{\delta}{\delta\theta} \left[ \bar{\mathcal{B}}_{\mu\nu}^* \bar{\mathcal{B}}^{\mu\nu} + \bar{\mathcal{M}}_\mu^* \bar{\mathcal{M}}^\mu + \bar{\mathcal{M}}_\mu \bar{\mathcal{M}}^{\mu*} + \bar{\mathcal{N}}^* \bar{\mathcal{N}} + \bar{\mathcal{N}}^* \bar{\mathcal{N}} + \bar{\mathcal{O}}^* \bar{\mathcal{O}} \right. \\ & \left. + \bar{\mathcal{O}} \bar{\mathcal{O}}^* + \bar{\mathcal{T}}^* \bar{\mathcal{T}} + \bar{\mathcal{S}}_\mu^* \bar{\mathcal{S}}^\mu + \Gamma(x, \theta) \right].\end{aligned}\quad (3.33)$$

In VSR framework also, we observe that using equations of motion of auxiliary fields and ghost fields of the shift symmetry, this effective Lagrangian density reduces to the original BRST invariant Lagrangian density.

### 3.3 VSR modified extended anti-BRST invariant BV action

In this subsection, we discuss the VSR modified extended anti-BRST invariant BV action for the Abelian rank-2 antisymmetric tensor field. First of all, let us write the VSR modified anti-BRST symmetry transformation ( $s_{ab}$ ), which leaves the Lagrangian density (3.5) for the 2-form gauge theory invariant, as follows,

$$\begin{aligned}s_{ab}B_{\mu\nu} = & \partial_\mu \tilde{\rho}_\nu - \partial_\nu \tilde{\rho}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\rho}_\mu, \\ s_{ab}\tilde{\rho}_\mu = & -i \left( \partial_\mu - \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) \tilde{\sigma}, \quad s_{ab}\tilde{\sigma} = 0, \quad s_{ab}\rho_\mu = -i\beta_\mu, \\ s_{ab}\beta_\mu = & 0, \quad s_{ab}\sigma = \chi, \quad s_{ab}\chi = 0, \quad s_{ab}\varphi = -\tilde{\chi}, \quad s_{ab}\tilde{\chi} = 0.\end{aligned}\quad (3.34)$$

This VSR modified anti-BRST transformation is nilpotent and plays an important role in defining physical unitarity. However, this transformation does not anticommute with

the BRST transformation (3.6) in absolute fashion, i.e.  $\{s_b, s_{ab}\} \neq 0$  for some fields. One should not bother for this in real sense as the absolutely anticommutativity can be achieved on ground of Curci-Ferrari (CF) type restriction. This is emphasized in the next section (in case of Abelian 3-form gauge theory) with more details.

The VSR modified gauge-fixed fermion corresponding to the anti-BRST transformation,  $\bar{\Psi}$ , is defined by

$$\begin{aligned} \bar{\Psi} = i \left[ \rho_\nu (\partial_\mu B^{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B^{\mu\nu} + k_1 \beta^\nu) - \sigma \partial_\mu \tilde{\rho}^\mu + \sigma \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}^\mu \right. \\ \left. + \varphi (\partial_\mu \rho^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \rho^\mu + k_2 \chi) \right]. \end{aligned} \quad (3.35)$$

As the the gauge-fixing part of the Lagrangian density is anti-BRST exact and, thus, can be written in terms of anti-BRST variation of  $\bar{\Psi}$  as following:

$$\mathcal{L}_{gf} = s_{ab} \bar{\Psi}. \quad (3.36)$$

To discuss the extended anti-BRST symmetry within VSR framework, we do follow the same procedure as in the case of BRST transformation. Thus, here we demand that extended anti-BRST operation on  $(\Phi - \bar{\Phi})$  should have same structure of the original anti-BRST transformations with shifted fields. This requirement leads to the following VSR modified extended anti-BRST transformations:

$$\begin{aligned} s_{ab} \bar{B}_{\mu\nu} &= B_{\mu\nu}^*, \quad s_{ab} B_{\mu\nu} = B_{\mu\nu}^* + (\partial_\mu \tilde{\rho}_\nu - \partial_\mu \bar{\rho}_\nu - \partial_\nu \tilde{\rho}_\mu + \partial_\nu \bar{\rho}_\mu - \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\rho}_\nu \\ &\quad + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\rho}_\nu + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\rho}_\mu - \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{\rho}_\mu), \quad s_{ab} \bar{\rho}_\mu = \rho_\mu^*, \\ s_{ab} \bar{\rho}_\mu &= \tilde{\rho}_\mu^*, \quad s_{ab} \tilde{\rho}_\mu = \bar{\rho}_\mu^* - i \partial_\mu \tilde{\sigma} + i \partial_\mu \bar{\sigma} + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \sigma - \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\sigma}, \\ s_{ab} \rho_\mu &= \rho_\mu^* - i \beta_\mu + i \bar{\beta}_\mu, \quad s_{ab} \bar{\sigma} = \tilde{\sigma}^*, \quad s_{ab} \tilde{\sigma} = \bar{\sigma}^*, \quad s_{ab} \bar{\beta}_\mu = \beta_\mu^*, \\ s_{ab} \beta_\mu &= \beta_\mu^*, \quad s_{ab} \bar{\sigma} = \sigma^*, \quad s_{ab} \sigma = \sigma^* - \chi + \bar{\chi}, \quad s_{ab} \bar{\chi} = \chi^*, \\ s_{ab} \chi &= \chi^*, \quad s_{ab} \bar{\varphi} = \varphi^*, \quad s_{ab} \varphi = \varphi^* - \tilde{\chi} + \bar{\chi}, \quad s_{ab} \tilde{\chi} = \tilde{\chi}^*, \quad s_{ab} \tilde{\chi} = \tilde{\chi}^*. \end{aligned} \quad (3.37)$$

The antifields  $B_{\mu\nu}^*, \tilde{\rho}_\mu^*, \rho_\mu^*, \tilde{\sigma}^*, \beta_\mu^*, \psi, \sigma^*, \chi^*, \varphi^*$  and  $\tilde{\chi}^*$  do not vary under extended anti-BRST transformations as the transformations are nilpotent in nature. Moreover, the ghost fields of the shift symmetry transform under VSR modified extended anti-BRST transformations as follows,

$$\begin{aligned} s_{ab} \psi_{\mu\nu} &= L_{\mu\nu}, \quad s_{ab} \epsilon_\mu = M_\mu, \quad s_{ab} \xi_\mu = \bar{M}_\mu, \quad s_{ab} \epsilon = N, \\ s_{ab} \psi &= \bar{N}, \quad s_{ab} \eta_\mu = S_\mu, \quad s_{ab} \Sigma = O, \quad s_{ab} \eta = \bar{O}, \quad s_{ab} \phi = T, \\ s_{ab} \bar{M}_\mu &= O, \quad s_{ab} L_{\mu\nu} = 0, \quad s_{ab} M_\mu = 0, \quad s_{ab} N = 0, \quad s_{ab} \bar{N} = 0, \\ s_{ab} S_\mu &= 0, \quad s_{ab} O = 0, \quad s_{ab} \bar{O} = 0, \quad s_{ab} \bar{T}. \end{aligned} \quad (3.38)$$

The transformations (3.37) and (3.38) together leads to complete extended anti-BRST transformations in VSR framework, which leave the shifted effective Lagrangian density invariant. With the help of these set of extended anti-BRST transformation, it is straight-forward to construct the superspace having extra fermion coordinate  $\bar{\theta}$  along with  $x_\mu$ .

### 3.4 VSR modified extended BRST and anti-BRST invariant superspace formulation

In this subsection, we develop a superspace formulation for VSR modified 2-form gauge theory which is manifestly invariant under the both extended BRST and extended anti-BRST transformations. To define a superspace for such theory, we need two Grassmannian coordinates,  $\theta$  and  $\bar{\theta}$ , together with  $x_\mu$ . Therefore, the superfields here depend on superspace  $(x_\mu, \theta, \bar{\theta})$ . Within VSR framework, the “super connection” 2-form  $(\omega^{(2)})$  and the field strength  $(F^{(3)})$ , respectively, are

$$\begin{aligned} \omega^{(2)} = & \frac{1}{2!} \mathcal{B}_{\mu\nu}(x, \theta, \bar{\theta})(dx^\mu \wedge dx^\nu) + \mathcal{M}_\mu(x, \theta, \bar{\theta})(dx^\mu \wedge d\theta) + \mathcal{N}(x, \theta, \bar{\theta})(d\theta \wedge d\theta) \\ & + \tilde{\mathcal{M}}_\mu(x, \bar{\theta}, \bar{\theta})(dx^\mu \wedge d\bar{\theta}) + \tilde{\mathcal{N}}(x, \bar{\theta}, \bar{\theta})(d\bar{\theta} \wedge d\bar{\theta}) + \mathcal{T}(x, \bar{\theta}, \bar{\theta})(d\bar{\theta} \wedge d\bar{\theta}), \end{aligned} \quad (3.39)$$

$$F^{(3)} = d\omega^{(2)}. \quad (3.40)$$

Here, the exterior derivative  $d$  has the following form:

$$d = dx^\mu \left( \partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \right) + d\theta \left( \partial_\theta - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\theta \right) + d\bar{\theta} \left( \partial_{\bar{\theta}} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_{\bar{\theta}} \right). \quad (3.41)$$

The components of the superfields can be computed by requiring the field strength to vanish along the directions of  $\theta$  and  $\bar{\theta}$ . The explicit expressions for these superfields are calculated in (A.1).

Exploiting the expressions of superfields given in (A.1), we compute the following expressions:

$$\begin{aligned} \frac{1}{2} \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \bar{\mathcal{B}}_{\mu\nu} \bar{\mathcal{B}}^{\mu\nu} = & L_{\mu\nu} \bar{B}^{\mu\nu} - B_{\mu\nu}^* \left( \psi^{\mu\nu} \partial^\mu \rho^\nu + \partial^\mu \rho^\nu + \partial^\nu \rho^\mu - \partial^\nu \bar{\rho}^\mu - \psi^{\mu\nu} \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \rho^\nu \right. \\ & \left. - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \rho^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \rho^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{\rho}^\mu \right), \\ \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \bar{\mathcal{M}}_\mu \bar{\mathcal{M}}^\mu = & \bar{M}_\mu \bar{\rho}^\mu + \bar{\rho}_\mu^* \left( \epsilon^\mu + i\partial^\mu \sigma - i\partial^\mu \bar{\sigma} - \frac{i}{2} \frac{m^2}{n \cdot \partial} n^\mu \sigma + \frac{i}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{\sigma} \right) + M_\mu \bar{\rho}^\mu \\ & + \rho_\mu^* (\xi^\mu - i\beta^\mu + i\bar{\beta}^\mu), \\ \frac{1}{2} \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \bar{\mathcal{N}} \bar{\mathcal{N}} = & N \bar{\sigma} - \sigma^* \epsilon, \quad \frac{1}{2} \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \bar{\tilde{\mathcal{N}}} \bar{\tilde{\mathcal{N}}} = \bar{\mathcal{N}} \bar{\tilde{\sigma}} - \tilde{\sigma}^* (\psi - \tilde{\chi} + \bar{\chi}), \\ \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \bar{\mathcal{O}} \bar{\mathcal{O}} = & \bar{O} \bar{\chi} + \tilde{\chi}^* \Sigma + O \bar{\chi} + \chi^* \eta, \quad \frac{1}{2} \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \bar{\mathcal{T}} \bar{\mathcal{T}} = T \bar{\varphi} - \varphi^* (\phi - \chi + \bar{\chi}), \\ \frac{1}{2} \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \bar{\mathcal{S}}_\mu \bar{\mathcal{S}}^\mu = & S_\mu \bar{\beta}^\mu - \beta_\mu^* \eta^\mu. \end{aligned} \quad (3.42)$$

By adding all the equations of above expression side by side, we get, remarkably, that this is nothing but the expression of Lagrangian density,  $\bar{\mathcal{L}}_{gf}$ , given in Eq. (3.16). Thus, we can write

$$\bar{\mathcal{L}}_{gf} = \frac{1}{2} \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \left[ \bar{\mathcal{B}}_{\mu\nu} \bar{\mathcal{B}}^{\mu\nu} + 2\bar{\mathcal{M}}_\mu \bar{\mathcal{M}}^\mu + \bar{\mathcal{N}} \bar{\mathcal{N}} + \bar{\tilde{\mathcal{N}}} \bar{\tilde{\mathcal{N}}} + 2\bar{\mathcal{O}} \bar{\mathcal{O}} + \bar{\mathcal{T}} \bar{\mathcal{T}} + \bar{\mathcal{S}}_\mu \bar{\mathcal{S}}^\mu \right]. \quad (3.43)$$

Eventually, we see that  $\bar{\mathcal{L}}_{gf}$  is nothing but the  $\theta\bar{\theta}$  component of the composite superfields. Therefore, this certifies the invariance of the  $\bar{\mathcal{L}}_{gf}$  under both the extended BRST and

extended anti-BRST transformations. The component form of super gauge-fixing fermion in this superspace is given by

$$\Gamma(x, \theta, \bar{\theta}) = \Psi + \theta s_b \Psi + \bar{\theta} s_{ab} \Psi + \theta \bar{\theta} s_b s_{ab} \Psi, \quad (3.44)$$

to express the  $\mathcal{L}_{gf}$  as  $\frac{\delta}{\delta \bar{\theta}} [\delta(\bar{\theta}) \Gamma(x, \theta, \bar{\theta})]$ . The  $\theta \bar{\theta}$  component of  $\Gamma(x, \theta, \bar{\theta})$  vanishes due to equations of motion in the theories having both BRST and anti-BRST invariance.

Therefore, the effective Lagrangian density (3.24), which is invariant under both the extended BRST and the extended anti-BRST transformations, can be expressed in superspace by

$$\begin{aligned} \mathcal{L}_{eff} = & \mathcal{L}_0(B_{\mu\nu} - \bar{B}_{\mu\nu}) + \frac{1}{2} \frac{\delta}{\delta \bar{\theta}} \frac{\delta}{\delta \theta} \left[ \bar{\mathcal{B}}_{\mu\nu} \bar{\mathcal{B}}^{\mu\nu} + 2 \bar{\mathcal{M}}_{\mu} \bar{\mathcal{M}}^{\mu} + \bar{\mathcal{N}} \bar{\mathcal{N}} + \bar{\tilde{\mathcal{N}}} \bar{\tilde{\mathcal{N}}} + 2 \bar{\mathcal{O}} \bar{\mathcal{O}} + \bar{\mathcal{T}} \bar{\mathcal{T}} + \bar{\mathcal{S}}_{\mu} \bar{\mathcal{S}}^{\mu} \right] \\ & + \frac{\delta}{\delta \theta} [\delta(\bar{\theta}) \Gamma(x, \theta, \bar{\theta})]. \end{aligned} \quad (3.45)$$

Here, we found that the gauge fixing parts of the effective Lagrangian density is the  $\theta$  components of the certain functional. Thus we observe that the VSR modified 2-form gauge theory in superspace described by two fermionic parameters also follow the same structure as the Lorentz invariant case.

#### 4 3-form gauge theory: VSR modified BV action in superspace

VSR generalization to the tensor field (reducible gauge) theories has also been done using a BV formulation [7, 8]. A rigorous construction of quantum field theory in VSR framework is also studied [45]. In this connection, we would like study the BV action tensor field of rank-3 in superspace. In particular, we generalize our previous results for the case of Abelian 3-form gauge theory which is relevant as it plays a crucial role to study the excitations of the quantized versions of strings, superstrings and related extended objects. The classical Lagrangian density for the Abelian 3-form gauge theory in VSR is given by [7, 8], as

$$\mathcal{L}_0 = \frac{1}{24} \tilde{F}_{\mu\nu\eta\xi} \tilde{F}^{\mu\nu\eta\xi}, \quad (4.1)$$

where the 4-form field strength tensor ( $\tilde{F}_{\mu\nu\eta\xi}$ ) has the following form:

$$\begin{aligned} \tilde{F}_{\mu\nu\eta\xi} = & \partial_{\mu} B_{\nu\eta\xi} - \partial_{\nu} B_{\eta\xi\mu} + \partial_{\eta} B_{\xi\mu\nu} - \partial_{\xi} B_{\mu\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_{\mu} B_{\nu\eta\xi} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_{\nu} B_{\eta\xi\mu} \\ & - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_{\eta} B_{\xi\mu\nu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_{\xi} B_{\mu\nu\eta}. \end{aligned} \quad (4.2)$$

Here  $B_{\mu\nu\eta}$  is totally antisymmetric rank-3 tensor gauge field and  $n_{\mu}$  is a constant null vector. The Lagrangian density (4.1) is not invariant under standard gauge transformation as the Lorentz invariance is broken by choosing an specific direction. However, this Lagrangian density is invariant under the following VSR modified gauge transformation

$$\delta B_{\mu\nu\eta} = \partial_{\mu} \lambda_{\nu\eta} + \partial_{\nu} \lambda_{\eta\mu} + \partial_{\eta} \lambda_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_{\mu} \lambda_{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_{\nu} \lambda_{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_{\eta} \lambda_{\mu\nu}, \quad (4.3)$$

where  $\lambda_{\mu\nu}$  is an arbitrary antisymmetric parameter. As this is a (VSR modified) gauge invariant theory, it contains redundant degrees of freedom. From the expression of (4.3), it is evident that the theory is reducible. Therefore, to quantize this theory correctly we need to fix the gauge appropriately. The gauge-fixed Lagrangian density in VSR is calculated by

$$\begin{aligned}
\mathcal{L}_{gf}^B = & \partial_\mu B^{\mu\nu\eta} B_{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B^{\mu\nu\eta} B_{\nu\eta} + \frac{1}{2} B_{\mu\nu} \tilde{B}^{\mu\nu} + \partial_\mu \tilde{c}_{\nu\eta} \partial^\mu c^{\nu\eta} + m^2 \tilde{c}_{\nu\eta} c^{\nu\eta} \\
& + \partial_\nu \tilde{c}_{\eta\mu} \partial^\mu c^{\nu\eta} - \partial_\nu \tilde{c}_{\eta\mu} \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} - \partial^\mu c^{\nu\eta} \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{c}^{\eta\mu} + n_\nu \tilde{c}^{\eta\mu} \frac{1}{4} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} \\
& + \partial_\mu \tilde{c}_{\mu\nu} \partial^\mu c^{\nu\eta} - \partial_\mu \tilde{c}_{\mu\nu} \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} - \partial^\mu c^{\nu\eta} \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \tilde{c}_{\mu\nu} + n_\eta \tilde{c}^{\mu\nu} \frac{1}{4} \frac{m^2}{n \cdot \partial} n_\mu \tilde{c}_{\mu\nu} \\
& - \partial_\mu \tilde{\beta}_\nu \partial^\mu \beta^\nu - m^2 \tilde{\beta}_\nu \beta^\nu + \partial_\nu \tilde{\beta}_\mu \partial^\mu \beta^\nu - \partial_\nu \tilde{\beta}_\mu \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \beta^\nu - \partial^\mu \beta^\nu \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\beta}_\mu \\
& + \frac{1}{4} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\beta}_\mu n^\mu \beta^\nu - B B_2 - \frac{1}{2} B_1^2 + \partial_\mu \tilde{c}^{\mu\nu} f_\nu - \partial_\mu c^{\mu\nu} \tilde{F}_\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{c}^{\mu\nu} f_\nu \\
& + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c^{\mu\nu} \tilde{F}_\nu + \partial_\mu \tilde{c}_2 \partial^\mu c_2 + m^2 \tilde{c}_2 c_2 + \tilde{f}_\mu f_\mu - \tilde{F}_\mu F^\mu + \partial_\mu \beta^\mu B_2 \\
& - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \beta^\mu B_2 + \partial_\mu \phi^\mu B_1 - \partial_\mu \tilde{\beta}^\mu B - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \phi^\mu B_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\beta}^\mu B. \quad (4.4)
\end{aligned}$$

Here, keeping the reducible nature of the theory, we have introduced extra auxiliary and ghost fields. For instance, antisymmetric ghost and antighost fields  $c_{\mu\nu}$  and  $\bar{c}_{\mu\nu}$  are Grassmannian and the vector field  $\phi_\mu$ , antisymmetric auxiliary fields  $B_{\mu\nu}, \tilde{B}_{\mu\nu}$  and auxiliary fields  $B, B_1, B_2$  are bosonic. The fields  $\beta_\mu$  and  $\tilde{\beta}_\mu$  are ghost of ghosts and are bosonic in nature. However,  $c_2$  and  $\bar{c}_2$  are ghost of ghost of ghosts with fermionic nature. The rest of the Grassmannian fields, i.e.,  $c_1, \bar{c}_1, f_\mu$  and  $\tilde{F}_\mu$  are auxiliary fields. It has been shown in Ref. [8] that the ghosts  $c_{\mu\nu}$  and  $\bar{c}_{\mu\nu}$ , ghost of ghosts  $\beta_\mu$  and  $\tilde{\beta}_\mu$  and ghost of ghost of ghosts  $c_2$  and  $\bar{c}_2$ , are massive.

To write the absolutely anticommuting BRST and anti-BRST invariant BV action for 3-form gauge theory in VSR, we consider an equivalent candidate to the above gauge-fixing Lagrangian density as following:

$$\begin{aligned}
\mathcal{L}_{gf}^{\tilde{B}} = & -\partial_\mu B^{\mu\nu\eta} \tilde{B}_{\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B^{\mu\nu\eta} \tilde{B}_{\nu\eta} + \frac{1}{2} B_{\mu\nu} \tilde{B}^{\mu\nu} + \partial_\mu \tilde{c}_{\nu\eta} \partial^\mu c^{\nu\eta} + m^2 \tilde{c}_{\nu\eta} c^{\nu\eta} \\
& + \partial_\nu \tilde{c}_{\eta\mu} \partial^\mu c^{\nu\eta} - \partial_\nu \tilde{c}_{\eta\mu} \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} - \partial^\mu c^{\nu\eta} \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{c}^{\eta\mu} + n_\nu \tilde{c}^{\eta\mu} \frac{1}{4} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} \\
& + \partial_\mu \tilde{c}_{\mu\nu} \partial^\mu c^{\nu\eta} - \partial_\mu \tilde{c}_{\mu\nu} \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} - \partial^\mu c^{\nu\eta} \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \tilde{c}_{\mu\nu} + n_\eta \tilde{c}^{\mu\nu} \frac{1}{4} \frac{m^2}{n \cdot \partial} n_\mu \tilde{c}_{\mu\nu} \\
& - \partial_\mu \tilde{\beta}_\nu \partial^\mu \beta^\nu - m^2 \tilde{\beta}_\nu \beta^\nu + \partial_\nu \tilde{\beta}_\mu \partial^\mu \beta^\nu - \partial_\nu \tilde{\beta}_\mu \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \beta^\nu - \partial^\mu \beta^\nu \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\beta}_\mu \\
& + \frac{1}{4} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\beta}_\mu n^\mu \beta^\nu - B B_2 - \frac{1}{2} B_1^2 + \partial_\mu c^{\mu\nu} \tilde{f}_\nu - \partial_\mu \tilde{c}^{\mu\nu} F_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{c}^{\mu\nu} F_\nu \\
& - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c^{\mu\nu} \tilde{f}_\nu + \partial_\mu \tilde{c}_2 \partial^\mu c_2 + m^2 \tilde{c}_2 c_2 + \tilde{f}_\mu f_\mu - \tilde{F}_\mu F^\mu + \partial_\mu \beta^\mu B_2 \\
& - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \beta^\mu B_2 + \partial_\mu \phi^\mu B_1 - \partial_\mu \tilde{\beta}^\mu B - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \phi^\mu B_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\beta}^\mu B. \quad (4.5)
\end{aligned}$$

These two Lagrangian densities are equivalent as they describe same dynamics of the theory on the following CF type restricted surface:

$$\begin{aligned} f_\mu + F_\mu &= \partial_\mu c_1 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c_1, \quad \bar{f}_\mu + \bar{F}_\mu = \partial_\mu \bar{c}_1 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{c}_1, \\ B_{\mu\nu} + \bar{B}_{\mu\nu} &= \partial_\mu \phi_\nu - \partial_\nu \phi_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \phi_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \phi_\mu. \end{aligned} \quad (4.6)$$

The VSR modified BRST transformations which leave the effective Lagrangian density,  $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{gf}^B$ , invariant are given by

$$\begin{aligned} s_b B_{\mu\nu\eta} &= (\partial_\mu c_{\nu\eta} + \partial_\nu c_{\eta\mu} + \partial_\eta c_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c_{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu c_{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta c_{\mu\nu}), \\ s_b \tilde{c}_{\mu\nu} &= B_{\mu\nu}, \quad s_b B_{\mu\nu} = \partial_\mu f_\nu - \partial_\nu f_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu f_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu f_\mu, \quad s_b \tilde{\beta}_\mu = \tilde{F}_\mu, \\ s_b \beta_\mu &= (\partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu) c_2, \quad s_b F_\mu = -(\partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu) B, \\ s_b \tilde{f}_\mu &= (\partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu) B_1, \quad s_b \tilde{c}_2 = B_2, \quad s_b c_1 = -B, \quad s_b \phi_\mu = f_\mu, \\ s_b \tilde{c}_1 &= B_1, \quad s_b [c_2, f_\mu, \tilde{F}_\mu, B, B_1, B_2, B_{\mu\nu}] = 0. \end{aligned} \quad (4.7)$$

Since the gauge fixed Lagrangian density  $\mathcal{L}_{gf}^B$  is BRST exact, so it can be written in terms of BRST variation of a gauge-fixing fermion  $\Psi$  as

$$\mathcal{L}_{gf}^B = s_b \Psi, \quad (4.8)$$

where the explicit form of  $\Psi$  is as following:

$$\begin{aligned} \Psi &= -\frac{1}{2} \tilde{c}_2 B + \frac{1}{2} B_2 c_1 - \frac{1}{2} \tilde{c}_1 B_1 - \frac{1}{2} (\partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\beta}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\beta}_\mu) c^{\mu\nu} \\ &+ \frac{1}{2} \tilde{c}_{\mu\nu} \tilde{B}^{\mu\nu} - \partial_\mu \tilde{c}_2 \beta^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} (n \cdot \beta) \tilde{c}_2 - \tilde{\beta}_\mu F^\mu - \phi_\mu \tilde{f}^\mu - \frac{1}{3} B_{\mu\nu\eta} (\partial^\mu \tilde{c}^{\nu\eta} + \partial^\nu \tilde{c}^{\eta\mu} \\ &+ \partial^\eta \tilde{c}^{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \tilde{c}^{\mu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu c^{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\eta \tilde{c}^{\mu\nu}). \end{aligned} \quad (4.9)$$

Here, one can see the VSR modification in the gauge-fixing fermion clearly. The anti-BRST symmetry transformations ( $s_{ab}$ ) for the above Lagrangian densities are given by

$$\begin{aligned} s_{ab} B_{\mu\nu\eta} &= (\partial_\mu \tilde{c}_{\nu\eta} + \partial_\nu \tilde{c}_{\eta\mu} + \partial_\eta \tilde{c}_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{c}_{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{c}_{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \tilde{c}_{\mu\nu}) \\ s_{ab} \tilde{c}_{\mu\nu} &= \partial_\mu \tilde{\beta}_\nu - \partial_\nu \tilde{\beta}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\beta}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\beta}_\mu, \quad s_{ab} c_{\mu\nu} = \tilde{B}_{\mu\nu}, \\ s_{ab} B_{\mu\nu} &= \partial_\mu \tilde{f}_\nu - \partial_\nu \tilde{f}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{f}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{f}_\mu, \quad s_{ab} \beta_\mu = F_\mu, \\ s_{ab} \tilde{\beta}_\mu &= \partial_\mu \tilde{c}_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{c}_2, \quad s_{ab} \tilde{F}_\mu = -\partial_\mu B_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_2, \\ s_{ab} f_\mu &= -\partial_\mu B_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_1, \quad s_{ab} c_2 = B, \quad s_{ab} c_1 = -B_1, \quad s_{ab} \phi_\mu = \tilde{f}_\mu, \\ s_{ab} \tilde{c}_1 &= -B_2, \quad s_{ab} [\tilde{c}_2, \tilde{f}_\mu, F_\mu, B, B_1, B_2, \tilde{B}_{\mu\nu}] = 0. \end{aligned} \quad (4.10)$$

Since both the BRST and anti-BRST transformations are absolutely anticommuting in nature, so both the BRST and anti-BRST exact Lagrangian densities can be expressed as follows,

$$\mathcal{L}_{gf}^B = s_b s_{ab} \left[ \frac{1}{2} \tilde{c}_2 c_2 - \frac{1}{2} \tilde{c}_1 c_1 - \frac{1}{2} \tilde{c}_{\mu\nu} c^{\mu\nu} - \tilde{\beta}_\mu \beta^\mu - \frac{1}{2} \phi_\mu \phi^\mu - \frac{1}{6} B_{\mu\nu\eta} B^{\mu\nu\eta} \right]. \quad (4.11)$$

Now, we will study the construction of VSR modified extended BRST transformation for Abelian 3-form gauge theory in next subsection.

#### 4.1 VSR modified extended BRST invariant BV action

Following the previous sections, we generalize the VSR modified extended BRST construction for the case of Abelian 3-form gauge theory in VSR. In this regard, we shift all the fields from their original values as follows

$$\begin{aligned} B_{\mu\nu\eta} &\rightarrow B_{\mu\nu\eta} - \bar{B}_{\mu\nu\eta}, \quad c_{\mu\nu} \rightarrow c_{\mu\nu} - \bar{c}_{\mu\nu}, \quad \tilde{c}_{\mu\nu} \rightarrow \tilde{c}_{\mu\nu} - \bar{\tilde{c}}_{\mu\nu}, \quad B_{\mu\nu} \rightarrow B_{\mu\nu} - \bar{B}_{\mu\nu}, \\ \tilde{B}_{\mu\nu} &\rightarrow \tilde{B}_{\mu\nu} - \bar{\tilde{B}}_{\mu\nu}, \quad \beta_\mu \rightarrow \beta_\mu - \bar{\beta}_\mu, \quad \tilde{\beta}_\mu \rightarrow \tilde{\beta}_\mu - \bar{\tilde{\beta}}_\mu, \quad F_\mu \rightarrow F_\mu - \bar{F}_\mu, \\ \tilde{F}_\mu &\rightarrow \tilde{F}_\mu - \bar{\tilde{F}}_\mu, \quad f_\mu \rightarrow f_\mu - \bar{f}_\mu, \quad \tilde{f}_\mu \rightarrow \tilde{f}_\mu - \bar{\tilde{f}}_\mu, \quad c_2 \rightarrow c_2 - \bar{c}_2, \\ \tilde{c}_2 &\rightarrow \tilde{c}_2 - \bar{\tilde{c}}_2, \quad c_1 \rightarrow c_1 - \bar{c}_1, \quad \tilde{c}_1 \rightarrow \tilde{c}_1 - \bar{\tilde{c}}_1, \quad \phi_\mu \rightarrow \phi_\mu - \bar{\phi}_\mu, \\ B &\rightarrow B - \bar{B}, \quad B_1 \rightarrow B_1 - \bar{B}_1, \quad B_2 \rightarrow B_2 - \bar{B}_2. \end{aligned} \quad (4.12)$$

Following the above shifts in fields, the effective Lagrangian density of the theory is modified by

$$\begin{aligned} \bar{\mathcal{L}} &= \mathcal{L}(B_{\mu\nu\eta} - \bar{B}_{\mu\nu\eta}, c_{\mu\nu} - \bar{c}_{\mu\nu}, \tilde{c}_{\mu\nu} - \bar{\tilde{c}}_{\mu\nu}, B_{\mu\nu} - \bar{B}_{\mu\nu}, \tilde{B}_{\mu\nu} - \bar{\tilde{B}}_{\mu\nu}, \beta_\mu - \bar{\beta}_\mu, \tilde{\beta}_\mu - \bar{\tilde{\beta}}_\mu, \\ &\quad F_\mu - \bar{F}_\mu, \tilde{F}_\mu - \bar{\tilde{F}}_\mu, f_\mu - \bar{f}_\mu, \tilde{f}_\mu - \bar{\tilde{f}}_\mu, c_2 - \bar{c}_2, \tilde{c}_2 - \bar{\tilde{c}}_2, c_1 - \bar{c}_1, \tilde{c}_1 - \bar{\tilde{c}}_1, \phi_\mu - \bar{\phi}_\mu, \\ &\quad B - \bar{B}, B_1 - \bar{B}_1, B_2 - \bar{B}_2). \end{aligned} \quad (4.13)$$

This Lagrangian density remains invariant under the BRST transformation (4.7) corresponding to the shifted fields. In fact, it is invariant under the following extended BRST transformations of fields:

$$s_b \Phi(x) = \alpha(x), \quad s_b \bar{\Phi}(x) = \alpha(x) - \beta(x), \quad (4.14)$$

where  $\Phi(x)$  and  $\bar{\Phi}(x)$  represent the collective original and shifted fields respectively. Here  $\beta(x)$  refers the original BRST transformation of with respect to the shifted fields, whereas  $\alpha(x)$  corresponds the ghost fields associated with shift symmetry collectively. The explicit form of the extended BRST transformation (4.14) is given explicitly in Eq. (B.2) of Appendix B. Now, in order to make theory unchanged, we need to remove the contribution of these ghosts from the physical states. To do so, we further introduce the following anti-fields (anti-ghosts):  $B_{\mu\nu\eta}^*, c_{\mu\nu}^*, \tilde{c}_{\mu\nu}^*, B_{\mu\nu}^*, \tilde{B}_{\mu\nu}^*, \eta_\mu^*, \tilde{\beta}_\mu^*, F_\mu^*, \tilde{F}_\mu^*, f_\mu^*, \tilde{f}_\mu^*, c_2^*, \tilde{c}_2^*, c_1^*, \tilde{c}_1^*, \phi_\mu^*, B^*, B_1^*, B_2^*$  to the theory. The BRST variation of these antifields are given in (B.3). Now, to remove the redundancies, we fix the gauge for shift symmetry, which is achieved by adding



the following gauge fixing term to the VSR quantum action

$$\begin{aligned}
\bar{\mathcal{L}}_{gf}^B = & l_{\mu\nu\eta}\bar{B}^{\mu\nu\eta} - B_{\mu\nu\eta}^*(L^{\mu\nu\eta} - \partial^\mu c^{\nu\eta} + \partial^\mu \bar{c}^{\nu\eta} - \partial^\nu c^{\eta\mu} + \partial^\nu \bar{c}^{\eta\mu} - \partial^\eta c^{\mu\nu} + \partial^\eta \bar{c}^{\mu\nu} \\
& + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{c}^{\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu c^{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{c}^{\eta\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\eta c^{\mu\nu} \\
& - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\eta \bar{c}^{\mu\nu}) + \tilde{c}_{\mu\nu}^*(M^{\mu\nu} - \partial^\mu \beta^\nu + \partial^\mu \bar{\beta}^\nu + \partial^\nu \beta^\mu - \partial^\nu \bar{\beta}^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \beta^\nu \\
& - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{\beta}^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \beta^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{\beta}^\mu) + m_{\mu\nu} \tilde{c}^{\mu\nu} + c_{\mu\nu}^*(\tilde{M}^{\mu\nu} - B^{\mu\nu} + \bar{B}^{\mu\nu}) \\
& + n_{\mu\nu} \bar{B}^{\mu\nu} - B_{\mu\nu}^* N^{\mu\nu} + \bar{n}_{\mu\nu} \tilde{B}^{\mu\nu} - \tilde{B}_{\mu\nu}^*(\tilde{N}^{\mu\nu} - \partial^\mu F^\nu + \partial^\mu \bar{F}^\nu + \partial^\nu F^\mu \\
& - \partial^\nu \bar{F}^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu F^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{F}^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu F^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{F}^\mu) + o_\mu \bar{\beta}^\mu \\
& - \beta_\mu^*(O^\mu - \partial^\mu c_2 + \partial^\mu \bar{c}_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{c}_2) + \bar{o}_\mu \tilde{\beta}^\mu - \tilde{\beta}_\mu^*(\tilde{O}^\mu - \tilde{F}^\mu \\
& + \tilde{\bar{F}}^\mu) + \bar{p}_\mu \bar{F}^\mu + \tilde{F}_\mu^*(P^\mu + \partial^\mu B - \partial^\mu \bar{B} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{B}) + p_\mu \tilde{\bar{F}}^\mu \\
& + F_\mu^* \tilde{P}^\mu + \bar{q}_\mu \bar{f}^\mu + \tilde{f}_\mu^* Q^\mu + q_\mu \tilde{\bar{f}}^\mu + f_\mu^*(\tilde{Q}^\mu - \partial^\mu B_1 + \partial^\mu \bar{B}_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B_1 \\
& - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{B}_1) + \bar{r} \bar{c}_2 + \tilde{c}_2^* \mathfrak{R} + r \bar{\tilde{c}}_2 + c_2^*(\tilde{\mathfrak{R}} - B_2 + \bar{B}_2) + \bar{s} \bar{c}_1 + s \bar{\tilde{c}}_1 + v \bar{B}_1 \\
& + \tilde{c}_1^*(\mathfrak{S} + B - \bar{B}) + c_1^*(\tilde{\mathfrak{S}} - B_1 + \bar{B}_1) + t_\mu \bar{\phi}^\mu - \phi_\mu^*(T^\mu - f^\mu + \bar{f}^\mu) + u \bar{B} \\
& - B^* \mathbb{U} + w \bar{B}_2 - B_2^* \mathfrak{W} - B_1^* \mathfrak{V}.
\end{aligned} \tag{4.15}$$

where the fields  $L_{\mu\nu\eta}$ ,  $M_{\mu\nu}$ ,  $\tilde{M}_{\mu\nu}$ ,  $N_{\mu\nu}$ ,  $\tilde{N}_{\mu\nu}$ ,  $O_\mu$ ,  $\tilde{O}_\mu$ ,  $P_\mu$ ,  $\tilde{P}_\mu$ ,  $Q_\mu$ ,  $\tilde{Q}_\mu$ ,  $\mathfrak{R}$ ,  $\tilde{\mathfrak{R}}$ ,  $\mathfrak{S}$ ,  $\tilde{\mathfrak{S}}$ ,  $T_\mu$ ,  $\mathbb{U}$ ,  $\mathfrak{V}$  and  $\mathfrak{W}$  are the ghost fields associated with the shift symmetries for fields  $B_{\mu\nu\eta}$ ,  $c_{\mu\nu}$ ,  $\tilde{c}_{\mu\nu}$ ,  $B_{\mu\nu}$ ,  $\tilde{B}_{\mu\nu}$ ,  $\beta_\mu$ ,  $\tilde{\beta}_\mu$ ,  $F_\mu$ ,  $\tilde{F}_\mu$ ,  $f_\mu$ ,  $\tilde{f}_\mu$ ,  $c_2$ ,  $\tilde{c}_2$ ,  $c_1$ ,  $\tilde{c}_1$ ,  $\phi_\mu$ ,  $B$ ,  $B_1$ ,  $B_2$  respectively and the fields  $l_{\mu\nu\eta}$ ,  $m_{\mu\nu}$ ,  $\tilde{m}_{\mu\nu}$ ,  $n_{\mu\nu}$ ,  $\bar{n}_{\mu\nu}$ ,  $o_\mu$ ,  $\bar{o}_\mu$ ,  $p_\mu$ ,  $\bar{p}_\mu$ ,  $q_\mu$ ,  $\bar{q}_\mu$ ,  $r$ ,  $\bar{r}$ ,  $s$ ,  $\bar{s}$ ,  $t_\mu$ ,  $u$ ,  $v$ ,  $w$  are the Nakanishi-Lautrup type auxiliary fields corresponding to the antighost fields  $B_{\mu\nu\eta}^*$ ,  $c_{\mu\nu}^*$ ,  $\tilde{c}_{\mu\nu}^*$ ,  $B_{\mu\nu}^*$ ,  $\tilde{B}_{\mu\nu}^*$ ,  $\eta_\mu^*$ ,  $\tilde{\beta}_\mu^*$ ,  $F_\mu^*$ ,  $\tilde{F}_\mu^*$ ,  $f_\mu^*$ ,  $\tilde{f}_\mu^*$ ,  $c_2^*$ ,  $\tilde{c}_2^*$ ,  $c_1^*$ ,  $\tilde{c}_1^*$ ,  $\phi_\mu^*$ ,  $B^*$ ,  $B_1^*$ ,  $B_2^*$ , respectively.

The gauge-fixed Lagrangian density corresponding to shift symmetry,  $\bar{\mathcal{L}}_{gf}^B$ , is invariant under the extended BRST symmetry transformations given in Eqs. (B.2) and (B.3) of Appendix B. If we perform the equations of motion for auxiliary fields, then all bar fields

disappear and we left with the following gauge-fixed Lagrangian density:

$$\begin{aligned}
\bar{\mathcal{L}}_{gf}^B = & -B_{\mu\nu\eta}^*(L^{\mu\nu\eta} - \partial^\mu c^{\nu\eta} - \partial^\nu c^{\eta\mu} - \partial^\eta c^{\mu\nu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu c^{\eta\mu} \\
& + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\eta c^{\mu\nu}) + \tilde{c}_{\mu\nu}^*(M^{\mu\nu} - \tilde{\partial}^\mu \beta^\nu + \tilde{\partial}^\nu \beta^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \beta^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \beta^\mu) \\
& + c_{\mu\nu}^*(\tilde{M}^{\mu\nu} - B^{\mu\nu}) - B_{\mu\nu}^* N^{\mu\nu} + \tilde{B}_{\mu\nu}^*(\tilde{N}^{\mu\nu} - \partial^\mu f^\nu + \partial^\nu f^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu f^\nu \\
& - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu f^\mu) - \beta_\mu^*(O^\mu - \partial^\mu c_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c_2) + \tilde{\beta}_\mu^*(\tilde{O}^\mu - \tilde{F}^\mu) \\
& + \tilde{F}_\mu^*(P^\mu + \partial^\mu B - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B) + F_\mu^* \tilde{P}^\mu + \tilde{f}_\mu^* Q^\mu + f_\mu^*(\tilde{Q}^\mu - \partial^\mu B_1 \\
& + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B_1) + \tilde{c}_2^* \mathfrak{R} + c_2^*(\tilde{\mathfrak{R}} - B_2) + \tilde{c}_1^*(\mathfrak{S} + B) + c_1^*(\tilde{\mathfrak{S}} - B_1) \\
& - \phi_\mu^*(T^\mu - f^\mu) - B^* \mathbb{U} - B_1^* \mathfrak{V} - B_2^* \mathfrak{W}.
\end{aligned} \tag{4.16}$$

Furthermore, if the general gauge-fixing fermion of the Abelian 3-form gauge theory in VSR,  $\Psi$ , depends on the original fields only, then we can write a general gauge-fixed Lagrangian density for this theory with the original BRST symmetry in terms of  $\Psi$  as

$$\mathcal{L}_{gf}^B = s_b \Psi[\Phi] = \sum_{\Phi} (s_b \Phi) \frac{\delta \Psi}{\delta \Phi}, \tag{4.17}$$

where  $\Phi$  is the generic notation for all fields in the theory. Keeping the nature of fields (i.e fermionic and bosonic) in mind the above gauge-fixed Lagrangian density in can be expressed as

$$\begin{aligned}
\mathcal{L}_{gf}^B = & -\frac{\delta \Psi}{\delta B_{\mu\nu\eta}} L_{\mu\nu\eta} + \frac{\delta \Psi}{\delta c_{\mu\nu}} M_{\mu\nu} + \frac{\delta \Psi}{\delta \tilde{c}_{\mu\nu}} \tilde{M}_{\mu\nu} - \frac{\delta \Psi}{\delta B_{\mu\nu}} N_{\mu\nu} - \frac{\delta \Psi}{\delta \tilde{B}_{\mu\nu}} \tilde{N}_{\mu\nu} \\
& - \frac{\delta \Psi}{\delta \beta_\mu} O_\mu - \frac{\delta \Psi}{\delta \tilde{\beta}_\mu} \tilde{O}_\mu + \frac{\delta \Psi}{\delta F_\mu} P_\mu + \frac{\delta \Psi}{\delta \tilde{F}_\mu} \tilde{P}_\mu + \frac{\delta \Psi}{\delta f_\mu} Q_\mu + \frac{\delta \Psi}{\delta \tilde{f}_\mu} \tilde{Q}_\mu \\
& + \frac{\delta \Psi}{\delta c_2} \mathfrak{R} + \frac{\delta \Psi}{\delta \tilde{c}_2} \tilde{\mathfrak{R}} + \frac{\delta \Psi}{\delta c_1} \mathfrak{S} + \frac{\delta \Psi}{\delta \tilde{c}_1} \tilde{\mathfrak{S}} - \frac{\delta \Psi}{\delta \phi_\mu} T_\mu - \frac{\delta \Psi}{\delta B} \mathbb{U} - \frac{\delta \Psi}{\delta B_1} \mathfrak{V} - \frac{\delta \Psi}{\delta B_2} \mathfrak{W}.
\end{aligned} \tag{4.18}$$

Now, we are able to define the BV action for Abelian 3-form gauge theory in VSR by combining Eqs. (4.4), (4.16) and (4.18) as follows

$$\mathcal{L}_{eff} = \mathcal{L}_0(B_{\mu\nu\rho} - \bar{B}_{\mu\nu\rho}) + \mathcal{L}_{gf}^B + \bar{\mathcal{L}}_{gf}^B. \tag{4.19}$$

The explicit expression of  $\mathcal{L}_{eff}$  is given in (B.1) (see in Appendix B).

To obtain the identifications on the antifields of 3-form theory in VSR, we integrate the ghosts associated with the shift symmetry and utilize the gauge-fixing fermion (4.9).

We thus obtain the explicit values of antifields as following:

$$\begin{aligned}
B^{\mu\nu\eta\star} &= \frac{1}{3}(\partial^\mu \tilde{c}^{\nu\eta} + \partial^\nu \tilde{c}^{\eta\mu} + \partial^\eta \tilde{c}^{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \tilde{c}^{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \tilde{c}^{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\eta \tilde{c}^{\mu\nu}), \\
c^{\mu\nu\star} &= -\frac{1}{2} B^{\mu\nu}, \quad B^{\mu\nu\star} = -\frac{1}{2} \tilde{c}^{\mu\nu}, \quad \tilde{\beta}^{\mu\star} = F^\mu + \partial_\nu c^{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu c^{\mu\nu}, \\
\tilde{c}^{\mu\nu\star} &= \frac{1}{2}(\partial^\mu \tilde{\beta}^\nu - \partial^\nu \tilde{\beta}^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \tilde{\beta}^\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \tilde{\beta}^\mu) - \partial_\eta B^{\mu\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta B^{\mu\nu\eta}, \\
\beta^{\mu\star} &= -\frac{1}{2} \partial^\mu \tilde{c}_2 + \frac{1}{4} \frac{m^2}{n \cdot \partial} n^\mu \tilde{c}_2, \quad c_2^\star = \frac{1}{2} B - (\partial_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu) \beta^\mu, \quad \tilde{c}_1^\star = -\frac{1}{2} B_2, \\
\tilde{F}^{\mu\star} &= \tilde{\beta}^\mu, \quad f^{\mu\star} = \phi^\mu, \quad c_1^\star = \frac{1}{2} B_1, \quad \phi^{\mu\star} = \tilde{f}^\mu, \quad B^\star = \frac{1}{2} \tilde{c}_2, \quad B_1^\star = \frac{1}{2} \tilde{c}_1, \quad B_2^\star = -\frac{1}{2} c_1, \\
[\tilde{B}^{\mu\nu\star}, F^{\mu\star}, f^{\mu\star}, \tilde{c}_2^\star] &= 0.
\end{aligned} \tag{4.20}$$

Now, we are able to express gauge-fixed Lagrangian density corresponding to extended BRST transformations in terms of generalized gauge-fixing fermion as follows,

$$\begin{aligned}
\mathcal{L}_{gf}^B + \tilde{\mathcal{L}}_{gf}^B &= s_b \left( B_{\mu\nu\eta}^\star \bar{B}^{\mu\nu\eta} + c_{\mu\nu}^\star \bar{\tilde{c}}^{\mu\nu} + \tilde{c}_{\mu\nu}^\star \bar{c}^{\mu\nu} + B_{\mu\nu}^\star \bar{B}^{\mu\nu} + \beta_\mu^\star \bar{\beta}^\mu + \tilde{\beta}_\mu^\star \bar{\tilde{\beta}}^\mu + F_\mu^\star \bar{\tilde{F}}^\mu + \tilde{F}_\mu^\star \bar{F}^\mu \right. \\
&\quad \left. + f_\mu^\star \bar{\tilde{f}}^\mu + \tilde{f}_\mu^\star \bar{f}^\mu + c_2^\star \bar{\tilde{c}}_2 + \tilde{c}_2^\star \bar{c}_2 + c_1^\star \bar{\tilde{c}}_1 + \tilde{c}_1^\star \bar{c}_1 + \phi_\mu^\star \bar{\phi}^\mu + B^\star \bar{B} + B_1^\star \bar{B}_1 + B_2^\star \bar{B}_2 \right), \\
&=: s_b (\Phi^\star \bar{\Phi}),
\end{aligned} \tag{4.21}$$

where the collective fields  $\Phi$  and  $\bar{\Phi}$  denote the original fields and corresponding shifted fields respectively. Here the ghost number of the expression  $\Phi^\star \bar{\Phi} = -1$ , as we expect. Here we see a difference in the expression of generalized gauge-fixing fermion to that of the ordinary gauge-fixing fermion

$$\Psi = -[B_{\mu\nu\eta} B^{\mu\nu\eta\star} + \tilde{c}_{\mu\nu} c^{\mu\nu\star} + \tilde{\beta}_\mu \tilde{\beta}^{\mu\star} + \phi_\mu \phi^{\mu\star} + \tilde{c}_2 c_2^\star + \tilde{c}_1 c_1^\star + B_2 B_2^\star]. \tag{4.22}$$

Plugging the values of the antifields (4.20), we can recover the original Lagrangian density of 3-form gauge theory in VSR.

## 4.2 VSR modified extended BRST invariant BV action in superspace

Furthermore, while the BRST and the anti-BRST symmetries of 3-form theories can be given a geometrical meaning and have led to a superspace formulation of such theories [43], a superspace description of the VSR modified BV action does not exist so far. Here, we develop a superspace formalism of the VSR modified 3-form theory having extended BRST invariance only. To study the theory with extended BRST invariance only, we need one extra fermionic parameter  $\theta$  together with  $x_\mu$ . The components of superfields,  $\mathfrak{T}$ , in terms of generic fields  $\Phi$  is given by

$$\mathfrak{T}(x, \theta) = \Phi(x) + \theta(s_b \Phi). \tag{4.23}$$

With the help of extended BRST transformation (B.2), the explicit expressions for the superfield are listed in Eq. (B.4) of Appendix.

The gauge-fixed Lagrangian density corresponding to shift symmetry mentioned in (4.15) in the superspace is simplified as

$$\begin{aligned}\bar{\mathcal{L}}_{gf}^B &= \frac{\delta}{\delta\theta} \left[ \bar{\mathcal{B}}_{\mu\nu\eta}^* \bar{\mathcal{B}}^{\mu\nu\eta} + \bar{\mathcal{C}}_{\mu\nu}^* \bar{\mathcal{C}}^{\mu\nu} + \bar{\mathcal{C}}_{\mu\nu} \bar{\mathcal{C}}^{\mu\nu*} + \bar{\mathcal{B}}_{\mu\nu}^* \bar{\mathcal{B}}^{\mu\nu} + \bar{\mathcal{B}}_{\mu}^* \bar{\mathcal{B}}^{\mu} + \bar{\mathcal{F}}_{\mu}^* \bar{\mathcal{F}}^{\mu} + \bar{\mathcal{F}}_{\mu} \bar{\mathcal{F}}^{\mu*} \right. \\ &\quad \left. + \bar{\mathcal{F}}_{\mu} \bar{\mathcal{F}}^{\mu*} + \bar{\mathcal{F}}_{\mu} \bar{\mathcal{F}}^{\mu*} + \bar{\mathcal{C}}_1^* \bar{\mathcal{C}}_1 + \bar{\mathcal{C}}_1 \bar{\mathcal{C}}_1^* + \bar{\mathcal{C}}_2^* \bar{\mathcal{C}}_2 + \bar{\mathcal{C}}_2 \bar{\mathcal{C}}_2^* + \bar{\mathcal{B}}^* \bar{\mathcal{B}} + \bar{\mathcal{B}}_1^* \bar{\mathcal{B}}_1 + \bar{\mathcal{B}}_2^* \bar{\mathcal{B}}_2 \right], \\ &= \frac{\delta}{\delta\theta} [\bar{\mathcal{T}}^* \bar{\mathcal{T}}].\end{aligned}\quad (4.24)$$

Being the  $\theta$  component of a superfields,  $\bar{\mathcal{L}}_{gf}^B$  is invariant under the extended BRST transformation. In the extended BRST invariant superspace, the gauge-fixed fermion of the original VSR modified 3-form gauge theory in component form translates as

$$\Gamma(x, \theta) = \Psi(x) + \theta s_b \Psi. \quad (4.25)$$

If we assume a general gauge-fixed fermion  $\Psi$  depending on all the original fields, then a  $\Gamma(x, \theta)$  reads,

$$\begin{aligned}\Gamma(x, \theta) &= \Psi(x) + \theta \left[ -\frac{\delta\Psi}{\delta B_{\mu\nu\eta}} L_{\mu\nu\eta} + \frac{\delta\Psi}{\delta c_{\mu\nu}} M_{\mu\nu} + \frac{\delta\Psi}{\delta \bar{c}_{\mu\nu}} \tilde{M}_{\mu\nu} - \frac{\delta\Psi}{\delta B_{\mu\nu}} N_{\mu\nu} - \frac{\delta\Psi}{\delta \bar{B}_{\mu\nu}} \tilde{N}_{\mu\nu} \right. \\ &\quad - \frac{\delta\Psi}{\delta \beta_{\mu}} O_{\mu} - \frac{\delta\Psi}{\delta \bar{\beta}_{\mu}} \tilde{O}_{\mu} + \frac{\delta\Psi}{\delta F_{\mu}} P_{\mu} + \frac{\delta\Psi}{\delta \tilde{F}_{\mu}} \tilde{P}_{\mu} + \frac{\delta\Psi}{\delta f_{\mu}} Q_{\mu} + \frac{\delta\Psi}{\delta \tilde{f}_{\mu}} \tilde{Q}_{\mu} + \frac{\delta\Psi}{\delta c_2} \mathfrak{R} \\ &\quad \left. + \frac{\delta\Psi}{\delta \bar{c}_2} \tilde{\mathfrak{R}} + \frac{\delta\Psi}{\delta c_1} \mathfrak{S} + \frac{\delta\Psi}{\delta \bar{c}_1} \tilde{\mathfrak{S}} - \frac{\delta\Psi}{\delta \phi_{\mu}} T_{\mu} - \frac{\delta\Psi}{\delta B} \mathbb{U} - \frac{\delta\Psi}{\delta B_1} \mathfrak{V} - \frac{\delta\Psi}{\delta B_2} \mathfrak{W} \right].\end{aligned}\quad (4.26)$$

Therefore, the VSR modified gauge-fixing Lagrangian density (4.4) in the superspace formalism takes very compact form as

$$\mathcal{L}_{gf}^B = \frac{\delta\Gamma(x, \theta)}{\delta\theta}. \quad (4.27)$$

Here we note that the invariance of this Lagrangian density is obvious under the extended BRST transformation as it belongs to  $\theta$  component. Now, we are able to write the extended BRST invariant BV action in the superspace as follows

$$\mathcal{L}_{eff} = \mathcal{L}_0(B_{\mu\nu\rho} - \bar{B}_{\mu\nu\rho}) + \frac{\delta\Gamma(x, \theta)}{\delta\theta} + \frac{\delta}{\delta\theta} [\bar{\mathcal{T}}^* \bar{\mathcal{T}}]. \quad (4.28)$$

Here we observe that eliminating auxiliary fields and ghost fields of the shift symmetry through using equations of motion, this effective Lagrangian density reduces to the original BRST invariant Lagrangian density of 3-form gauge theory in VSR also.

### 4.3 VSR modified extended anti-BRST invariant BV action

In this subsection, we discuss the extended anti-BRST symmetry for Abelian 3-form gauge theory in VSR and their superspace description.

As the gauge-fixing Lagrangian density is anti-BRST exact, so one can write this in terms of corresponding gauge-fixing fermion  $\bar{\Psi}$  as

$$\mathcal{L}_{gf}^B = s_{ab} \bar{\Psi}, \quad (4.29)$$

where  $\bar{\Psi}$  has the following expression:

$$\begin{aligned}\bar{\Psi} = & -\frac{1}{2}B_2c_2 + \frac{1}{2}B_1c_1 + \frac{1}{2}\tilde{c}_1B + \frac{1}{2}B_{\mu\nu}c^{\mu\nu} - \frac{1}{2}\tilde{c}_{\mu\nu}(\partial^\mu\beta^\nu - \partial^\nu\beta^\mu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n^\mu\beta^\nu \\ & + \frac{1}{2}\frac{m^2}{n\cdot\partial}n^\nu\beta^\mu) + \tilde{F}_\mu\beta^\mu + \beta_\mu\partial^\mu c_2 - \beta_\mu\frac{1}{2}\frac{m^2}{n\cdot\partial}n^\mu c_2 + \frac{1}{2}\phi_\mu f^\mu + \frac{1}{3}B_{\mu\nu\eta}(\partial^\mu c^{\nu\eta} \\ & + \partial^\nu c^{\eta\mu} + \partial^\eta c^{\mu\nu} - \frac{1}{2}\frac{m^2}{n\cdot\partial}n^\mu c^{\nu\eta} - \frac{1}{2}\frac{m^2}{n\cdot\partial}n^\nu c^{\eta\mu} - \frac{1}{2}\frac{m^2}{n\cdot\partial}n^\eta c^{\mu\nu}).\end{aligned}\quad (4.30)$$

Keeping the structure of (B.2) (given in Appendix B) in mind, we demand that anti-BRST transformation of shifted fields  $(\Phi - \bar{\Phi})$  coincides with the anti-BRST transformations of the original fields  $(\Phi)$  if all the bar fields vanish. This requirement leads to the following form of the (VSR modified) extended anti-BRST transformations:

$$\begin{aligned}s_{ab}\bar{B}_{\mu\nu\eta} &= B_{\mu\nu\eta}^*, \\ s_{ab}B_{\mu\nu\eta} &= B_{\mu\nu\eta}^* + \partial_\mu\tilde{c}_{\nu\eta} - \partial_\mu\bar{c}_{\nu\eta} + \partial_\nu\tilde{c}_{\eta\mu} - \partial_\nu\bar{c}_{\eta\mu} + \partial_\eta\tilde{c}_{\mu\nu} - \partial_\eta\bar{c}_{\mu\nu} - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\tilde{c}_{\nu\eta} \\ &\quad + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{c}_{\nu\eta} - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\tilde{c}_{\eta\mu} + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\bar{c}_{\eta\mu} - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\eta\tilde{c}_{\mu\nu} + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\eta\bar{c}_{\mu\nu}, \\ s_{ab}\tilde{c}_{\mu\nu} &= \tilde{c}_{\mu\nu}^*, \quad s_{ab}\tilde{c}_{\mu\nu} = \tilde{c}_{\mu\nu}^* + \partial_\mu\tilde{\beta}_\nu - \partial_\mu\bar{\beta}_\nu - \partial_\nu\tilde{\beta}_\mu + \partial_\nu\bar{\beta}_\mu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\tilde{\beta}_\nu \\ &\quad + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{\beta}_\nu + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\tilde{\beta}_\mu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\bar{\beta}_\mu, \quad s_{ab}\bar{c}_{\mu\nu} = c_{\mu\nu}^*, \\ s_{ab}c_{\mu\nu} &= c_{\mu\nu}^* + \tilde{B}_{\mu\nu} - \bar{B}_{\mu\nu}, \quad s_{ab}B_{\mu\nu} = B_{\mu\nu}^* + \partial_\mu\tilde{f}_\nu - \partial_\mu\bar{f}_\nu - \partial_\nu\tilde{f}_\mu + \partial_\nu\bar{f}_\mu \\ &\quad - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\tilde{f}_\nu + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{f}_\nu + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\tilde{f}_\mu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\bar{f}_\mu, \quad s_{ab}\phi_\mu = \phi_\mu^* + \tilde{f}_\mu - \bar{f}_\mu, \\ s_{ab}\bar{B}_{\mu\nu} &= B_{\mu\nu}^*, \quad s_{ab}\bar{\beta}_\mu = \beta_\mu^*, \quad s_{ab}\beta_\mu = \beta_\mu^* + F_\mu - \bar{F}_\mu, \quad s_{ab}\bar{\beta}_\mu = \tilde{\beta}_\mu^*, \\ s_{ab}\tilde{\beta}_\mu &= \tilde{\beta}_\mu^* + \partial_\mu\tilde{c}_2 - \partial_\mu\bar{c}_2 - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\tilde{c}_2 + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{c}_2, \quad s_{ab}\bar{F}_\mu = \tilde{F}_\mu^*, \\ s_{ab}\tilde{F}_\mu &= \tilde{F}_\mu^* - \tilde{\partial}_\mu B_2 + \tilde{\partial}_\mu\bar{B}_2 + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu B_2 - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{B}_2, \quad s_{ab}\bar{f}_\mu = f_\mu^*, \\ s_{ab}f_\mu &= f_\mu^* - \partial_\mu B_1 + \partial_\mu\bar{B}_1 + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu B_1 - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{B}_1, \quad s_{ab}\bar{c}_2 = c_2^*, \\ s_{ab}c_2 &= c_2^* + B - \bar{B}, \quad s_{ab}\bar{c}_1 = c_1^*, \quad s_{ab}c_1 = c_1^* - B_1 + \bar{B}_1, \quad s_{ab}\bar{\phi}_\mu = \phi_\mu^*, \\ s_{ab}\bar{c}_1 &= \tilde{c}_1^*, \quad s_{ab}\tilde{c}_1 = \tilde{c}_1^* - B_2 + \bar{B}_2, \quad s_{ab}\bar{c}_2 = \tilde{c}_2^*, \quad s_{ab}\tilde{c}_2 = \tilde{c}_2^*, \quad s_{ab}\bar{f}_\mu = \tilde{f}_\mu^*, \\ s_{ab}\tilde{f}_\mu &= \tilde{f}_\mu^*, \quad s_{ab}\bar{F}_\mu = F_\mu^*, \quad s_{ab}F_\mu = F_\mu^*, \quad s_{ab}\bar{B} = B^*, \quad s_{ab}B = B^*, \quad s_{ab}\bar{B}_1 = B_1^*, \\ s_{ab}B_1 &= B_1^*, \quad s_{ab}\bar{B}_2 = B_2^*, \quad s_{ab}B_2 = B_2^*, \quad s_{ab}\bar{B}_{\mu\nu} = \tilde{B}_{\mu\nu}^*, \quad s_{ab}\tilde{B}_{\mu\nu} = \tilde{B}_{\mu\nu}^*.\end{aligned}\quad (4.31)$$

However, the following fields:  $B_{\mu\nu\eta}^*$ ,  $\tilde{c}_{\mu\nu}^*$ ,  $c_{\mu\nu}^*$ ,  $B_{\mu\nu}^*$ ,  $\beta_\mu^*$ ,  $\tilde{\beta}_\mu^*$ ,  $\tilde{F}_\mu^*$ ,  $f_\mu^*$ ,  $c_2^*$ ,  $c_1^*$ ,  $\phi_\mu^*$ ,  $\tilde{c}_1^*$ ,  $\tilde{c}_2^*$ ,  $\tilde{f}_\mu^*$ ,  $F_\mu^*$ ,  $B^*$ ,  $B_1^*$ ,  $B_2^*$  and  $\tilde{B}_{\mu\nu}^*$ , do not change under extended anti-BRST transformation which ensures the nilpotency of the transformation. The ghost fields associated with the shift

symmetry change under the extended anti-BRST transformations as

$$\begin{aligned}
s_{ab}L_{\mu\nu\eta} &= l_{\mu\nu\eta}, \quad s_{ab}M_{\mu\nu} = m_{\mu\nu}, \quad s_{ab}\tilde{M}_{\mu\nu} = \tilde{m}_{\mu\nu}, \quad s_{ab}N_{\mu\nu} = n_{\mu\nu}, \\
s_{ab}\tilde{N}_{\mu\nu} &= \tilde{n}_{\mu\nu}, \quad s_{ab}O_\mu = o_\mu, \quad s_{ab}\tilde{O}_\mu = \tilde{o}_\mu, \quad s_{ab}P_\mu = p_\mu, \quad s_{ab}\tilde{P}_\mu = \tilde{p}_\mu, \\
s_{ab}Q_\mu &= q_\mu, \quad s_{ab}\tilde{Q}_\mu = \tilde{q}_\mu, \quad s_{ab}\mathfrak{R} = r, \quad s_{ab}\tilde{\mathfrak{R}} = \tilde{r}, \quad s_{ab}\mathfrak{S} = s, \\
s_{ab}\tilde{\mathfrak{S}} &= \tilde{s}, \quad s_{ab}T_\mu = t_\mu, \quad s_{ab}\mathbb{U} = u, \quad s_{ab}\mathfrak{V} = v, \quad s_{ab}\mathfrak{W} = w.
\end{aligned} \tag{4.32}$$

Rest of fields, whose transformation is written here, do not change under the extended anti-BRST transformation. To describe the superspace formulation of the Abelian 3-form gauge theory in VSR which has only extended anti-BRST invariance, we need one extra Grassmannian coordinate, say  $\bar{\theta}$ . It is now straightforward to define the superfields in this superspace which involve the extended anti-BRST transformations along the  $\bar{\theta}$  coordinates respectively. Therefore, the superspace description of Abelian 3-form gauge theory in VSR also holds.

#### 4.4 VSR modified extended BRST and anti-BRST invariant superspace formulation

In this section, we construct the superspace formulation for the Abelian 3-form gauge theory in VSR which is manifestly invariant under both the extended BRST transformations and the extended anti-BRST transformations. To describe such superspace, we introduce two extra Grassmann coordinates,  $\theta$  and  $\bar{\theta}$  together with  $x_\mu$ . Now, we compute the components of the superfields by requiring the field strength to vanish along the directions of  $\theta$  and  $\bar{\theta}$ . This leads to the superfields to have following form in this superspace generically:

$$\mathfrak{T}(x, \theta, \bar{\theta}) = \Phi(x) + \theta(s_b\Phi) + \bar{\theta}(s_{ab}\Phi) + \theta\bar{\theta}(s_b s_{ab}\Phi). \tag{4.33}$$

Here  $\mathfrak{T}$  and  $\Phi$  describe all the superfields and the fields generically. The explicit expression for the individual superfields can be found in Eq. (B.5) of the Appendix B.

Exploiting superfields (B.5) we compute the relations (B.6). Interestingly, we establish a relation between the gauge-fixing Lagrangian density corresponding to shift symmetry,  $\bar{\mathcal{L}}_{gf}^B$  (4.4), and composite superfields as follows,

$$\begin{aligned}
\bar{\mathcal{L}}_{gf}^B &= \frac{1}{2} \frac{\delta}{\delta\theta} \frac{\delta}{\delta\bar{\theta}} \left[ \bar{\mathcal{B}}_{\mu\nu\eta} \bar{\mathcal{B}}^{\mu\nu\eta} + 2\bar{\mathcal{C}}_{\mu\nu} \bar{\mathcal{C}}^{\mu\nu} + \bar{\mathcal{B}}_{\mu\nu} \bar{\mathcal{B}}^{\mu\nu} + \bar{\mathcal{B}}_{\mu\nu} \bar{\mathcal{B}}^{\mu\nu} + \bar{\mathcal{B}}_\mu \bar{\mathcal{B}}^\mu + \bar{\mathcal{B}}_\mu \bar{\mathcal{B}}^\mu \right. \\
&\quad \left. + 2\bar{\mathcal{F}}_\mu \bar{\mathcal{F}}^\mu + 2\bar{\mathfrak{F}}_\mu \bar{\mathfrak{F}}^\mu + 2\bar{\mathcal{C}}_2 \bar{\mathcal{C}}_2 + 2\bar{\mathcal{C}}_1 \bar{\mathcal{C}}_1 + \bar{\Phi}_\mu \bar{\Phi}^\mu + \bar{\mathcal{B}} \bar{\mathcal{B}} + \bar{\mathcal{B}}_1 \bar{\mathcal{B}}_1 + \bar{\mathcal{B}}_2 \bar{\mathcal{B}}_2 \right], \tag{4.34}
\end{aligned}$$

This relation manifests that the gauge-fixed Lagrangian density  $\bar{\mathcal{L}}_{gf}^B$  is invariant under both the extended BRST and extended anti-BRST transformations.

The super gauge-fixing fermion for the theory having both the extended BRST and extended anti-BRST invariance in superspace is given by

$$\Gamma(x, \theta, \bar{\theta}) = \Psi(x) + \theta s_b \Psi + \bar{\theta} s_{ab} \Psi + \theta\bar{\theta} s_b s_{ab} \Psi. \tag{4.35}$$

In general, all four components of the super gauge-fixing fermion will be non-trivial, implying that if we choose as  $\mathcal{L}_{gf}^B = s_b \Psi$ , then it will not be invariant under generalized

anti-BRST transformations. This follows from the fact that the last component of the super gauge-fixing fermion (4.35) is non-vanishing in general. However, if the gauge-fixed Lagrangian density in VSR is both extended BRST and anti-BRST invariant, then the  $\theta\bar{\theta}$  component of super gauge-fixing fermion would vanish, because when we use the equations of motion the bar fields vanish and the theory reduces to the original theory, where, by assumption, the gauge-fixed Lagrangian density in VSR is both extended BRST and anti-BRST invariant. Therefore, for an arbitrary super gauge-fixing fermion that leads to a BRST and anti-BRST invariant gauge-fixing Lagrangian density, one can choose

$$\mathcal{L}_{gf}^B = \frac{\delta}{\delta\theta} [\delta(\bar{\theta})\Gamma(x, \theta, \bar{\theta})] = s_b\Psi. \quad (4.36)$$

Now, the effective Lagrangian density (4.19) possessing both the extended BRST and anti-BRST symmetries in superspace can be expressed by

$$\begin{aligned} \mathcal{L}_{eff} = & \mathcal{L}_0(B_{\mu\nu\rho} - \bar{B}_{\mu\nu\rho}) + \frac{1}{2} \frac{\delta}{\delta\theta} \frac{\delta}{\delta\theta} \left[ \bar{B}_{\mu\nu\eta} \bar{B}^{\mu\nu\eta} + 2\bar{C}_{\mu\nu} \bar{C}^{\mu\nu} + \bar{\tilde{B}}_{\mu\nu} \bar{\tilde{B}}^{\mu\nu} + \bar{B}_{\mu\nu} \bar{B}^{\mu\nu} + \bar{B}_\mu \bar{B}^\mu \right. \\ & + \bar{\tilde{B}}_\mu \bar{\tilde{B}}^\mu + 2\bar{\mathcal{F}}_\mu \bar{\mathcal{F}}^\mu + 2\bar{\tilde{\mathcal{F}}}_\mu \bar{\tilde{\mathcal{F}}}^\mu + 2\bar{\tilde{C}}_2 \bar{\tilde{C}}_2 + 2\bar{\tilde{C}}_1 \bar{\tilde{C}}_1 + \bar{\Phi}_\mu \bar{\Phi}^\mu + \bar{B}\bar{B} + \bar{B}_1 \bar{B}_1 + \bar{B}_2 \bar{B}_2 \Big] \\ & + \frac{\delta}{\delta\theta} [\delta(\bar{\theta})\Gamma(x, \theta, \bar{\theta})]. \end{aligned} \quad (4.37)$$

Using the auxiliary fields equations of motion the bar fields can be set zero. However, integration of the ghost fields of the shift symmetry leads to the explicit expressions for the antifields, which, when substituted into the VSR modified Lagrangian density, yield the BV action. The superspace formulation of BV action for the VSR modified 3-form gauge theory has similar description as the Lorentz invariant case.

## 5 Concluding Remarks

In VSR, the spacetime translational symmetry is retained to preserve the energy-momentum and also the usual relativistic dispersion relation. Keeping the significance of VSR in mind, in this paper, we have discussed the VSR description of the non-Abelian 1-form, Abelian 2-form and Abelian 3-form gauge theories. We have constructed the extended BRST and anti-BRST transformation (which include a shift symmetry) for these theories. To fix the shift symmetry, we need the antighost fields which coincide with the antifields of the BV formulation for each gauge theories in VSR. Furthermore, we have formulated these VSR modified theories in superspace. First, we have found that the extended BRST invariant Lagrangian densities of  $p = 1, 2, 3$ -form gauge theories in VSR can be written manifestly in a superspace with one additional fermionic coordinate, i.e.,  $(x_\mu, \theta)$ . Similarly, extended anti-BRST invariant Lagrangian densities of these theories in VSR can be written manifestly in a superspace with coordinates  $(x_\mu, \bar{\theta})$  where  $\bar{\theta}$  is another fermionic coordinate. Finally, a superspace description of the (manifestly covariant) BV action of these theories in VSR having both the extended BRST and extended anti-BRST invariance requires two additional Grassmann coordinates  $(x_\mu, \theta, \bar{\theta})$ . In this context, we have noted that if the gauge-fixed Lagrangian density in VSR is both extended BRST and anti-BRST invariant,

then the  $\theta\bar{\theta}$  component of super gauge-fixing fermion would vanish, because when we use the equations of motion the bar fields would vanish and the theory reduces to the original theory, where, by assumption, the gauge-fixed Lagrangian density in VSR is both extended BRST and anti-BRST invariant. The structure of the results we obtained here by studying BV action of  $p = 1, 2, 3$ -forms gauge theories with preferred direction is not very different to that of Lorentz invariant case. Unlike the Lorentz invariant case, the novel observation is that in VSR scenario, all the fields and superfields acquire mass, which modifies the masses of the original dispersion relations. It will be interesting to extend this superspace formulation to some regularization procedure at one-loop order, where we believe that the superfield associated with the one-loop order term of the action may have the VSR modified anomalies and Wess-Zumino terms. Also, the extension of this superspace formulation for the more general cases in which VSR modified anomalies and Wess-Zumino terms depend on the antifields will be interesting to explore.

## Acknowledgments

We are thankful to Dr. Anton Ilderton for his careful reading of the manuscript and helpful comments.



## A Mathematical details of VSR modified Abelian 2-form gauge theory in superspace

The explicit component form of superfields for VSR modified Abelian 2-form gauge theory in superspace having both extended BRST and anti-BRST invariance are:

$$\begin{aligned}
\mathcal{B}_{\mu\nu}(x, \theta, \bar{\theta}) &= B_{\mu\nu}(x) + \theta\psi_{\mu\nu} + \bar{\theta}(B_{\mu\nu}^* + \partial_\mu\tilde{\rho}_\nu - \partial_\mu\bar{\tilde{\rho}}_\nu - \partial_\nu\tilde{\rho}_\mu + \partial_\nu\bar{\tilde{\rho}}_\mu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\tilde{\rho}_\nu \\
&\quad + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{\tilde{\rho}}_\nu + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\tilde{\rho}_\nu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\bar{\tilde{\rho}}_\mu) + \theta\bar{\theta}[L_{\mu\nu} + i(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu \\
&\quad - \partial_\mu\bar{\beta}_\nu + \partial_\nu\bar{\beta}_\mu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\beta_\nu + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\beta_\mu + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{\beta}_\nu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\bar{\beta}_\mu)], \\
\bar{\mathcal{B}}_{\mu\nu}(x, \theta, \bar{\theta}) &= \bar{B}_{\mu\nu}(x) + \theta(\psi_{\mu\nu} - \partial_\mu\rho_\nu + \partial_\mu\bar{\rho}_\nu + \partial_\nu\rho_\mu - \partial_\nu\bar{\rho}_\mu + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\rho_\nu \\
&\quad - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{\rho}_\nu - \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\rho_\mu + \frac{1}{2}\frac{m^2}{n\cdot\partial}n_\nu\bar{\rho}_\mu) + \bar{\theta}B_{\mu\nu}^* + \theta\bar{\theta}L_{\mu\nu}, \\
\mathcal{M}_\mu(x, \theta, \bar{\theta}) &= \rho_\mu(x) + \theta\epsilon_\mu + \bar{\theta}(\rho_\mu^* - i\beta_\mu + i\bar{\beta}_\mu) + \theta\bar{\theta}M_\mu, \\
\bar{\mathcal{M}}_\mu(x, \theta, \bar{\theta}) &= \bar{\rho}_\mu(x) + \theta(\epsilon_\mu - i\partial_\mu\sigma + i\partial_\mu\bar{\sigma} + \frac{i}{2}\frac{m^2}{n\cdot\partial}n_\mu\sigma - \frac{i}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{\sigma}) + \bar{\theta}\rho_\mu^* + \theta\bar{\theta}M_\mu, \\
\mathcal{N}(x, \theta, \bar{\theta}) &= \sigma(x) + \theta\epsilon + \bar{\theta}(\sigma^* + \chi - \bar{\chi}) + \theta\bar{\theta}N, \\
\bar{\mathcal{N}}(x, \theta, \bar{\theta}) &= \bar{\sigma}(x) + \theta\epsilon + \bar{\theta}\sigma^* + \theta\bar{\theta}N, \\
\tilde{\mathcal{M}}_\mu(x, \theta, \bar{\theta}) &= \tilde{\rho}_\mu(x) + \theta\xi_\mu + \bar{\theta}(\tilde{\rho}_\mu^* - i\partial_\mu\tilde{\sigma} + \frac{i}{2}\frac{m^2}{n\cdot\partial}n_\mu\tilde{\sigma} + i\partial_\mu\bar{\tilde{\sigma}} - \frac{i}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{\tilde{\sigma}}) \\
&\quad + \theta\bar{\theta}(\bar{M}_\mu - i\partial_\mu\tilde{\chi} + \frac{i}{2}\frac{m^2}{n\cdot\partial}n_\mu\tilde{\chi} + i\partial_\mu\bar{\tilde{\chi}} - \frac{i}{2}\frac{m^2}{n\cdot\partial}n_\mu\bar{\tilde{\chi}}), \\
\bar{\tilde{\mathcal{M}}}_\mu(x, \theta, \bar{\theta}) &= \bar{\tilde{\rho}}_\mu(x) + \theta(\xi_\mu - i\beta_\mu + i\bar{\beta}_\mu) + \bar{\theta}\tilde{\rho}_\mu^* + \theta\bar{\theta}\bar{M}_\mu, \\
\mathcal{S}_\mu(x, \theta, \bar{\theta}) &= \beta_\mu(x) + \theta\eta_\mu + \bar{\theta}\beta_\mu^* + \theta\bar{\theta}S_\mu, \\
\bar{\mathcal{S}}_\mu(x, \theta, \bar{\theta}) &= \bar{\beta}_\mu(x) + \theta\eta_\mu + \bar{\theta}\beta_\mu^* + \theta\bar{\theta}S_\mu, \\
\tilde{\mathcal{N}}(x, \theta, \bar{\theta}) &= \tilde{\sigma}(x) + \theta\psi + \bar{\theta}\tilde{\sigma}^* + \theta\bar{\theta}\tilde{N}, \\
\bar{\tilde{\mathcal{N}}}(x, \theta, \bar{\theta}) &= \bar{\tilde{\sigma}}(x) + \theta(\psi - \tilde{\chi} + \bar{\tilde{\psi}}) + \bar{\theta}\tilde{\sigma}^* + \theta\bar{\theta}\tilde{N}, \\
\mathcal{O}(x, \theta, \bar{\theta}) &= \chi(x) + \theta\Sigma + \bar{\theta}\chi^* + \theta\bar{\theta}O, \\
\bar{\mathcal{O}}(x, \theta, \bar{\theta}) &= \bar{\chi}(x) + \theta\Sigma + \bar{\theta}\chi^* + \theta\bar{\theta}O, \\
\tilde{\mathcal{O}}(x, \theta, \bar{\theta}) &= \tilde{\chi}(x) + \theta\eta + \bar{\theta}\tilde{\chi}^* + \theta\bar{\theta}\tilde{O}, \\
\bar{\tilde{\mathcal{O}}}(x, \theta, \bar{\theta}) &= \bar{\tilde{\chi}}(x) + \theta\eta + \bar{\theta}\tilde{\chi}^* + \theta\bar{\theta}\tilde{O}, \\
\mathcal{T}(x, \theta, \bar{\theta}) &= \varphi(x) + \theta\phi + \bar{\theta}(\varphi^* - \tilde{\chi} + \bar{\tilde{\chi}}) + \theta\bar{\theta}T, \\
\bar{\mathcal{T}}(x, \theta, \bar{\theta}) &= \bar{\varphi}(x) + \theta(\phi - \chi + \bar{\chi}) + \bar{\theta}\varphi^* + \theta\bar{\theta}T.
\end{aligned} \tag{A.1}$$

## B Mathematical details of VSR modified Abelian 3-form gauge theory

The explicit form of the BV action of VSR modified Abelian 3-form gauge theory is calculated as

$$\begin{aligned}
\mathcal{L}_{eff} = & \frac{1}{24} H_{\mu\nu\eta\xi} H^{\mu\nu\eta\xi} + B_{\mu\nu\eta}^* (\partial^\mu c^{\nu\eta} + \partial^\nu c^{\eta\mu} + \partial^\eta c^{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu c^{\eta\mu} \\
& - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\eta c^{\mu\nu}) - \tilde{c}_{\mu\nu}^* (\partial^\mu \beta^\nu - \partial^\nu \beta^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \beta^\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \beta^\mu) - c_{\mu\nu}^* B^{\mu\nu} \\
& + \tilde{B}_{\mu\nu}^* (\partial^\mu f^\nu - \partial^\nu f^\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu f^\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu f^\mu) + \beta_\mu^* \partial^\mu c_2 + \tilde{\beta}_\mu^* \tilde{F}^\mu + \tilde{F}_\mu^* \partial^\mu B \\
& - f_\mu^* \partial^\mu B_1 - \beta_\mu^* \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c_2 - \tilde{F}_\mu^* \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B + f_\mu^* \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B_1 - c_2^* B_2 + \tilde{c}_1^* B \\
& - c_1^* B_1 + \phi_\mu^* f^\mu - \left( B^{\mu\nu\eta\star} + \frac{\delta\Psi}{\delta B_{\mu\nu\eta}} \right) L_{\mu\nu\eta} + \left( \tilde{c}^{\mu\nu\star} + \frac{\delta\Psi}{\delta c_{\mu\nu}} \right) M_{\mu\nu} \\
& + \left( c^{\mu\nu\star} + \frac{\delta\Psi}{\delta \tilde{c}_{\mu\nu}} \right) \tilde{M}_{\mu\nu} - \left( B^{\mu\nu\star} + \frac{\delta\Psi}{\delta B_{\mu\nu}} \right) N_{\mu\nu} - \left( \tilde{B}^{\mu\nu\star} + \frac{\delta\Psi}{\delta \tilde{B}_{\mu\nu}} \right) \tilde{N}_{\mu\nu} \\
& - \left( \beta^{\mu\star} + \frac{\delta\Psi}{\delta \beta_\mu} \right) O_\mu - \left( \tilde{\beta}^{\mu\nu\star} + \frac{\delta\Psi}{\delta \tilde{\beta}_\mu} \right) \tilde{O}_\mu + \left( \tilde{F}^{\mu\star} + \frac{\delta\Psi}{\delta F_\mu} \right) P_\mu \\
& + \left( F^{\mu\star} + \frac{\delta\Psi}{\delta \tilde{F}_\mu} \right) \tilde{P}_\mu + \left( \tilde{f}^{\mu\star} + \frac{\delta\Psi}{\delta f_\mu} \right) Q_\mu + \left( f^{\mu\star} + \frac{\delta\Psi}{\delta \tilde{f}_\mu} \right) \tilde{Q}_\mu + \left( \tilde{c}_2^* + \frac{\delta\Psi}{\delta c_2} \right) \mathfrak{R} \\
& + \left( c_2^* + \frac{\delta\Psi}{\delta \tilde{c}_2} \right) \tilde{\mathfrak{R}} + \left( \tilde{c}_1^* + \frac{\delta\Psi}{\delta c_1} \right) \mathfrak{S} + \left( c_1^* + \frac{\delta\Psi}{\delta \tilde{c}_1} \right) \tilde{\mathfrak{S}} - \left( \phi^{\mu\star} + \frac{\delta\Psi}{\delta \phi_\mu} \right) T_\mu \\
& - \left( B^{\star} + \frac{\delta\Psi}{\delta B} \right) \mathbb{U} - \left( B_1^* + \frac{\delta\Psi}{\delta B_1} \right) \mathfrak{V} - \left( B_2^* + \frac{\delta\Psi}{\delta B_2} \right) \mathfrak{W}. \tag{B.1}
\end{aligned}$$

The explicit form of the VSR modified extended BRST transformations for the Abelian 3-form gauge theory is

$$\begin{aligned}
s_b B_{\mu\nu\eta} &= L_{\mu\nu\eta}, \\
s_b \bar{B}_{\mu\nu\eta} &= L_{\mu\nu\eta} - \left( \partial_\mu c_{\nu\eta} - \partial_\mu \bar{c}_{\nu\eta} + \partial_\nu c_{\eta\mu} - \partial_\nu \bar{c}_{\eta\mu} + \partial_\eta c_{\mu\nu} - \partial_\eta \bar{c}_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c_{\nu\eta} \right. \\
&\quad \left. + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{c}_{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu c_{\eta\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{c}_{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta c_{\mu\nu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \bar{c}_{\mu\nu} \right), \\
s_b c_{\mu\nu} &= M_{\mu\nu}, \quad s_b \tilde{c}_{\mu\nu} = \tilde{M}_{\mu\nu}, \quad s_b \bar{c}_{\mu\nu} = M_{\mu\nu} - (\partial_\mu \beta_\nu - \partial_\mu \bar{\beta}_\nu - \partial_\nu \beta_\mu + \partial_\nu \bar{\beta}_\mu \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \beta_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\beta}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \beta_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{\beta}_\mu), \\
s_b \tilde{\bar{c}}_{\mu\nu} &= \tilde{M}_{\mu\nu} - B_{\mu\nu} + \bar{B}_{\mu\nu}, \quad s_b B_{\mu\nu} = N_{\mu\nu}, \quad s_b \bar{B}_{\mu\nu} = N_{\mu\nu}, \quad s_b \tilde{B}_{\mu\nu} = \tilde{N}_{\mu\nu}, \\
s_b \tilde{\bar{B}}_{\mu\nu} &= \tilde{N}_{\mu\nu} - (\partial_\mu f_\nu - \partial_\mu \bar{f}_\nu - \partial_\nu f_\mu + \partial_\nu \bar{f}_\mu \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu f_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{f}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu f_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{f}_\mu), \\
s_b \beta_\mu &= O_\mu, \quad s_b \bar{\beta}_\mu = O_\mu - \partial_\mu c_2 + \partial_\mu \bar{c}_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{c}_2, \\
s_b \tilde{\beta}_\mu &= \tilde{O}_\mu, \quad s_b \tilde{\bar{\beta}}_\mu = \tilde{O}_\mu - \tilde{F}_\mu + \tilde{\bar{F}}_\mu, \quad s_b F_\mu = P_\mu, \\
s_b \bar{F}_\mu &= P_\mu + \partial_\mu B - \partial_\mu \bar{B} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}, \\
s_b \tilde{F}_\mu &= \tilde{P}_\mu, \quad s_b \tilde{\bar{F}}_\mu = \tilde{P}_\mu, \quad s_b f_\mu = Q_\mu, \quad s_b \bar{f}_\mu = Q_\mu, \quad s_b \tilde{f}_\mu = \tilde{Q}_\mu, \\
s_b \tilde{\bar{f}}_\mu &= \tilde{Q}_\mu - \partial_\mu B_1 + \partial_\mu \bar{B}_1 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_1, \\
s_b c_2 &= \mathfrak{R}, \quad s_b \bar{c}_2 = \mathfrak{R}, \quad s_b \tilde{c}_2 = \tilde{\mathfrak{R}}, \quad s_b \tilde{\bar{c}}_2 = \tilde{\mathfrak{R}} - B_2 + \bar{B}_2, \\
s_b c_1 &= \mathfrak{S}, \quad s_b \bar{c}_1 = \mathfrak{S} + B - \bar{B}, \quad s_b \tilde{c}_1 = \tilde{\mathfrak{S}}, \quad s_b \phi_\mu = T_\mu, \quad s_b \bar{\phi}_\mu = T_\mu - f_\mu + \bar{f}_\mu, \\
s_b \tilde{c}_1 &= \tilde{\mathfrak{S}} - B_1 + \bar{B}_1, \quad s_b B = \mathbb{U}, \quad s_b B_1 = \mathfrak{V}, \quad s_b \bar{B}_1 = \mathfrak{V}, \\
s_b \bar{B} &= \mathbb{U}, \quad s_b B_2 = \mathfrak{W}, \quad s_b \bar{B}_2 = \mathfrak{W}, \quad s_b \Omega = 0,
\end{aligned} \tag{B.2}$$

where  $\Omega \equiv [L_{\mu\nu\eta}, M_{\mu\nu}, \tilde{M}_{\mu\nu}, N_{\mu\nu}, \tilde{N}_{\mu\nu}, O_\mu, \tilde{O}_\mu, P_\mu, \tilde{P}_\mu, Q_\mu, \tilde{Q}_\mu, \mathfrak{R}, \tilde{\mathfrak{R}}, \mathfrak{S}, \tilde{\mathfrak{S}}, T_\mu, \mathbb{U}, \mathfrak{V}, \mathfrak{W}]$  are the ghosts corresponding to the shift symmetry.

The VSR modified BRST transformation of antifields are

$$\begin{aligned}
s_b B_{\mu\nu\eta}^* &= l_{\mu\nu\eta}, \quad s_b c_{\mu\nu}^* = m_{\mu\nu}, \quad s_b \tilde{c}_{\mu\nu}^* = \bar{m}_{\mu\nu}, \quad s_b B_{\mu\nu}^* = n_{\mu\nu}, \\
s_b \beta_\mu^* &= o_\mu, \quad s_b \tilde{\beta}_\mu^* = \bar{o}_\mu, \quad s_b F_\mu^* = p_\mu, \quad s_b \tilde{F}_\mu^* = \bar{p}_\mu, \quad s_b f_\mu^* = q_\mu, \\
s_b \tilde{f}_\mu^* &= \bar{q}_\mu, \quad s_b c_2^* = r, \quad s_b \tilde{c}_2^* = \bar{r}, \quad s_b c_1^* = s, \quad s_b \tilde{c}_1^* = \bar{s}, \\
s_b \phi_\mu^* &= t_\mu, \quad s_b B^* = u, \quad s_b B_1^* = v, \quad s_b B_2^* = w, \quad s_b \tilde{B}_{\mu\nu}^* = \bar{n}_{\mu\nu}, \quad s_b \Lambda = 0.
\end{aligned} \tag{B.3}$$

where  $\Lambda \equiv l_{\mu\nu\eta}, m_{\mu\nu}, \bar{m}_{\mu\nu}, n_{\mu\nu}, \bar{n}_{\mu\nu}, o_\mu, \bar{o}_\mu, p_\mu, \bar{p}_\mu, q_\mu, \bar{q}_\mu, r, \bar{r}, s, \bar{s}, t_\mu, u, v, w$  are the auxiliary fields.

The superfields and anti-superfields in component form for the VSR modified extended

BRST invariant 3-form theory are

$$\begin{aligned}
\mathcal{B}_{\mu\nu\eta}(x, \theta) &= B_{\mu\nu\eta}(x) + \theta L_{\mu\nu\eta}, \quad \mathcal{C}_{\mu\nu}(x, \theta) = c_{\mu\nu}(x) + \theta M_{\mu\nu}, \\
\bar{\mathcal{B}}_{\mu\nu\eta}(x, \theta) &= \bar{B}_{\mu\nu\eta}(x) + \theta \left[ L_{\mu\nu\eta} - (\partial_\mu c_{\nu\eta} - \partial_\mu \bar{c}_{\nu\eta} + \partial_\nu c_{\eta\mu} - \partial_\nu \bar{c}_{\eta\mu} + \partial_\eta c_{\mu\nu} - \partial_\eta \bar{c}_{\mu\nu} \right. \\
&\quad \left. - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c_{\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{c}_{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu c_{\eta\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{c}_{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta c_{\mu\nu} \right. \\
&\quad \left. + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \bar{c}_{\mu\nu} \right), \\
\bar{\mathcal{C}}_{\mu\nu}(x, \theta) &= \bar{c}_{\mu\nu}(x) + \theta \left[ M_{\mu\nu} - \left( \partial_\mu \beta_\nu - \partial_\mu \bar{\beta}_\nu - \partial_\nu \beta_\mu + \partial_\nu \bar{\beta}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \beta_\nu \right. \right. \\
&\quad \left. \left. + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\beta}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \beta_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{\beta}_\mu \right) \right], \\
\tilde{\mathcal{C}}_{\mu\nu}(x, \theta) &= \tilde{c}_{\mu\nu}(x) + \theta \tilde{M}_{\mu\nu}, \quad \bar{\tilde{\mathcal{C}}}_{\mu\nu}(x, \theta) = \bar{\tilde{c}}_{\mu\nu}(x) + \theta (\tilde{M}_{\mu\nu} - B_{\mu\nu} + \bar{B}_{\mu\nu}), \\
\mathcal{B}_{\mu\nu}(x, \theta) &= B_{\mu\nu}(x) + \theta N_{\mu\nu}, \quad \bar{\mathcal{B}}_{\mu\nu}(x, \theta) = \bar{B}_{\mu\nu}(x) + \theta N_{\mu\nu}, \\
\tilde{\mathcal{B}}_{\mu\nu}(x, \theta) &= \tilde{B}_{\mu\nu}(x) + \theta \tilde{N}_{\mu\nu}, \quad \mathcal{B}_\mu(x, \theta) = \beta_\mu(x) + \theta O_\mu, \\
\bar{\mathcal{B}}_\mu(x, \theta) &= \bar{B}_\mu(x) + \theta \left[ \tilde{N}_{\mu\nu} - (\partial_\mu f_\nu - \partial_\mu \bar{f}_\nu - \partial_\nu f_\mu + \partial_\nu \bar{f}_\mu \right. \\
&\quad \left. - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu f_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{f}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu f_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{f}_\mu \right), \\
\bar{\mathcal{B}}_\mu(x, \theta) &= \bar{\beta}_\mu(x) + \theta \left( O_\mu - \partial_\mu c_2 + \partial_\mu \bar{c}_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{c}_2 \right), \\
\tilde{\mathcal{B}}_\mu(x, \theta) &= \tilde{\beta}_\mu(x) + \theta \tilde{O}_\mu, \quad \bar{\tilde{\mathcal{B}}}_\mu(x, \theta) = \bar{\tilde{\beta}}_\mu(x) + \theta (\tilde{O}_\mu - \tilde{F}_\mu + \bar{\tilde{F}}_\mu), \\
\bar{\mathcal{F}}_\mu(x, \theta) &= \bar{F}_\mu(x) + \theta \left( P_\mu + \partial_\mu B - \partial_\mu \bar{B} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B} \right), \\
\tilde{\mathcal{F}}_\mu(x, \theta) &= \tilde{F}_\mu(x) + \theta \tilde{P}_\mu, \quad \bar{\tilde{\mathcal{F}}}_\mu(x, \theta) = \bar{\tilde{F}}_\mu(x) + \theta \tilde{P}_\mu, \quad \mathcal{F}_\mu(x, \theta) = F_\mu(x) + \theta P_\mu, \\
\mathfrak{F}_\mu(x, \theta) &= f_\mu(x) + \theta Q_\mu, \quad \bar{\mathfrak{F}}_\mu(x, \theta) = \bar{f}_\mu(x) + \theta Q_\mu, \quad \tilde{\mathfrak{F}}_\mu(x, \theta) = \tilde{f}_\mu(x) + \theta \tilde{Q}_\mu, \\
\bar{\tilde{\mathfrak{F}}}_\mu(x, \theta) &= \bar{\tilde{f}}_\mu(x) + \theta \left( \tilde{Q}_\mu - \partial_\mu B_1 + \partial_\mu \bar{B}_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_1 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_1 \right), \\
\mathcal{C}_2(x, \theta) &= c_2(x) + \theta \mathfrak{R}, \quad \bar{\mathcal{C}}_2(x, \theta) = \bar{c}_2(x) + \theta \mathfrak{R}, \quad \tilde{\mathcal{C}}_2(x, \theta) = \tilde{c}_2(x) + \theta \mathfrak{R}, \\
\bar{\tilde{\mathcal{C}}}_2(x, \theta) &= \bar{\tilde{c}}_2(x) + \theta (\mathfrak{R} - B_2 + \bar{B}_2), \quad \mathcal{C}_1(x, \theta) = c_1(x) + \theta \mathfrak{S}, \\
\tilde{\mathcal{C}}_1(x, \theta) &= \tilde{c}_1(x) + \theta \mathfrak{S}, \quad \bar{\tilde{\mathcal{C}}}_1(x, \theta) = \bar{\tilde{c}}_1(x) + \theta (\mathfrak{S} - B_1 + \bar{B}_1), \\
\Phi_\mu(x, \theta) &= \phi_\mu(x) + \theta T_\mu, \quad \bar{\Phi}_\mu(x, \theta) = \bar{\phi}_\mu(x) + \theta (T_\mu - f_\mu + \bar{f}_\mu), \\
\mathcal{B}(x, \theta) &= B(x) + \theta \mathbb{U}, \quad \bar{\mathcal{B}}(x, \theta) = \bar{B}(x) + \theta \mathbb{U}, \quad \mathcal{B}_1(x, \theta) = B_1(x) + \theta \mathfrak{V}, \\
\bar{\mathcal{B}}_1(x, \theta) &= \bar{B}_1(x) + \theta \mathfrak{V}, \quad \mathcal{B}_2(x, \theta) = B_2(x) + \theta \mathfrak{W}, \quad \bar{\mathcal{B}}_2(x, \theta) = \bar{B}_2(x) + \theta \mathfrak{W}, \\
\bar{\mathcal{B}}_{\mu\nu}^*(x, \theta) &= B_{\mu\nu}^*(x) + \theta L_{\mu\nu\eta}, \quad \bar{\mathcal{C}}_{\mu\nu}^*(x, \theta) = c_{\mu\nu}^*(x) + \theta M_{\mu\nu}, \quad \bar{\tilde{\mathcal{C}}}_{\mu\nu}^*(x, \theta) = \tilde{c}_{\mu\nu}^*(x) + \theta \tilde{M}_{\mu\nu}, \\
\bar{\mathcal{B}}_{\mu\nu}^*(x, \theta) &= B_{\mu\nu}^*(x) + \theta N_{\mu\nu}, \quad \bar{\tilde{\mathcal{B}}}_{\mu\nu}^*(x, \theta) = \tilde{B}_{\mu\nu}^*(x) + \theta \tilde{N}_{\mu\nu}, \quad \bar{\mathcal{B}}_\mu^*(x, \theta) = \beta_\mu^*(x) + \theta O_\mu, \\
\bar{\tilde{\mathcal{B}}}_\mu^*(x, \theta) &= \tilde{\beta}_\mu^*(x) + \theta \tilde{O}_\mu, \quad \bar{\mathcal{F}}_\mu^*(x, \theta) = F_\mu^*(x) + \theta P_\mu, \quad \bar{\tilde{\mathcal{F}}}_\mu^*(x, \theta) = \tilde{F}_\mu^*(x) + \theta \tilde{P}_\mu, \\
\bar{\mathfrak{F}}_\mu^*(x, \theta) &= f_\mu^*(x) + \theta Q_\mu, \quad \bar{\tilde{\mathfrak{F}}}_\mu^*(x, \theta) = \tilde{f}_\mu^*(x) + \theta \tilde{Q}_\mu, \quad \bar{\mathcal{C}}_2^*(x, \theta) = c_2^*(x) + \theta \mathfrak{R}, \\
\bar{\tilde{\mathcal{C}}}_2^*(x, \theta) &= \tilde{c}_2^*(x) + \theta \mathfrak{R}, \quad \bar{\mathcal{C}}_1^*(x, \theta) = c_1^*(x) + \theta \mathfrak{S}, \quad \bar{\tilde{\mathcal{C}}}_1^*(x, \theta) = \tilde{c}_1^*(x) + \theta \mathfrak{S}, \\
\bar{\mathcal{B}}_1^*(x, \theta) &= B_1^*(x) + \theta \mathbb{U}, \quad \bar{\mathcal{B}}_2^*(x, \theta) = B_2^*(x) + \theta \mathfrak{V}, \quad \bar{\mathcal{B}}_2^*(x, \theta) = B_2^*(x) + \theta \mathfrak{W}, \\
\bar{\mathcal{C}}_1^*(x, \theta) &= \bar{c}_1^*(x) + \theta (\mathfrak{S} + B - \bar{B}), \quad \bar{\Phi}_\mu^*(x, \theta) = \phi_\mu^*(x) + \theta T_\mu. \tag{B.4}
\end{aligned}$$

The superfields for both extended BRST and anti-BRST invariant 3-form theory are

$$\begin{aligned}
\mathcal{B}_{\mu\nu\eta}(x, \theta, \bar{\theta}) &= B_{\mu\nu\eta}(x) + \theta L_{\mu\nu\eta} + \bar{\theta}(B_{\mu\nu\eta}^* + \partial_\mu \tilde{c}_{\nu\eta} - \partial_\mu \bar{c}_{\nu\eta} + \partial_\nu \tilde{c}_{\eta\mu} - \partial_\nu \bar{c}_{\eta\mu} + \partial_\eta \tilde{c}_{\mu\nu} \\
&\quad - \partial_\eta \bar{c}_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{c}_{\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{c}_{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{c}_{\eta\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{c}_{\eta\mu} \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \tilde{c}_{\mu\nu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \bar{c}_{\mu\nu}) + \theta \bar{\theta}(l_{\mu\nu\eta} + \partial_\mu B_{\nu\eta} - \partial_\mu \bar{B}_{\nu\eta} + \partial_\nu B_{\eta\mu} \\
&\quad - \partial_\nu \bar{B}_{\eta\mu} + \partial_\eta B_{\mu\nu} - \partial_\eta \bar{B}_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_{\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_{\nu\eta} \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu B_{\eta\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{B}_{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta B_{\mu\nu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \bar{B}_{\mu\nu}), \\
\mathcal{C}_{\mu\nu}(x, \theta, \bar{\theta}) &= c_{\mu\nu}(x) + \theta M_{\mu\nu} + \bar{\theta}(c_{\mu\nu}^* + \tilde{B}_{\mu\nu} - \bar{B}_{\mu\nu}) + \theta \bar{\theta}(m_{\mu\nu} + \partial_\mu \bar{f}_\nu - \partial_\mu f_\nu - \partial_\nu f_\mu \\
&\quad + \partial_\nu \bar{f}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu f_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{f}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu f_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{f}_\mu), \\
\bar{\mathcal{B}}_{\mu\nu\eta}(x, \theta, \bar{\theta}) &= \bar{B}_{\mu\nu\eta}(x) + \theta(L_{\mu\nu\eta} - (\partial_\mu c_{\nu\eta} - \partial_\mu \bar{c}_{\nu\eta} + \partial_\nu c_{\eta\mu} - \partial_\nu \bar{c}_{\eta\mu} + \partial_\eta c_{\mu\nu} - \partial_\eta \bar{c}_{\mu\nu} \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c_{\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{c}_{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu c_{\eta\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{c}_{\eta\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta c_{\mu\nu} \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\eta \bar{c}_{\mu\nu})) + \bar{\theta} B_{\mu\nu\eta}^* + \theta \bar{\theta} l_{\mu\nu\eta}, \\
\bar{\mathcal{C}}_{\mu\nu}(x, \theta, \bar{\theta}) &= \bar{c}_{\mu\nu}(x) + \theta(M_{\mu\nu} - (\partial_\mu \beta_\nu - \partial_\mu \bar{\beta}_\nu - \partial_\nu \beta_\mu + \partial_\nu \bar{\beta}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \beta_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\beta}_\nu \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \beta_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{\beta}_\mu)) + \bar{\theta} c_{\mu\nu}^* + \theta \bar{\theta} m_{\mu\nu}, \\
\tilde{\mathcal{C}}_{\mu\nu}(x, \theta, \bar{\theta}) &= \tilde{c}_{\mu\nu}(x) + \theta \tilde{M}_{\mu\nu} + \bar{\theta}(\tilde{c}_{\mu\nu}^* + \partial_\mu \tilde{\beta}_\nu - \partial_\mu \bar{\tilde{\beta}}_\nu - \partial_\nu \tilde{\beta}_\mu + \partial_\nu \bar{\tilde{\beta}}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\beta}_\nu \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\tilde{\beta}}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{\beta}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{\tilde{\beta}}_\mu) + \theta \bar{\theta}(\tilde{m}_{\mu\nu} + \partial_\mu \tilde{F}_\nu - \partial_\mu \bar{\tilde{F}}_\nu \\
&\quad - \partial_\nu \tilde{F}_\mu + \partial_\nu \bar{\tilde{F}}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{F}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\tilde{F}}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{F}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{\tilde{F}}_\mu), \\
\tilde{\bar{\mathcal{C}}}_{\mu\nu}(x, \theta, \bar{\theta}) &= \tilde{\bar{c}}_{\mu\nu}(x) + \theta(\tilde{M}_{\mu\nu} - B_{\mu\nu} + \bar{B}_{\mu\nu}) + \bar{\theta} \tilde{c}_{\mu\nu}^* + \theta \bar{\theta} \tilde{m}_{\mu\nu}, \\
\mathcal{B}_{\mu\nu}(x, \theta, \bar{\theta}) &= B_{\mu\nu}(x) + \theta N_{\mu\nu} + \bar{\theta}(B_{\mu\nu}^* + \partial_\mu \tilde{f}_\nu - \partial_\mu \bar{\tilde{f}}_\nu - \partial_\nu \tilde{f}_\mu + \partial_\nu \bar{\tilde{f}}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{f}_\nu \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{\tilde{f}}_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \tilde{f}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{\tilde{f}}_\mu) + \theta \bar{\theta} n_{\mu\nu}, \\
\bar{\mathcal{B}}_{\mu\nu}(x, \theta, \bar{\theta}) &= \bar{B}_{\mu\nu}(x) + \theta N_{\mu\nu} + \bar{\theta} B_{\mu\nu}^* + \theta \bar{\theta} n_{\mu\nu}, \\
\tilde{\mathcal{B}}_{\mu\nu}(x, \theta, \bar{\theta}) &= \tilde{B}_{\mu\nu}(x) + \theta \tilde{N}_{\mu\nu} + \bar{\theta} \tilde{B}_{\mu\nu}^* + \theta \bar{\theta} \tilde{n}_{\mu\nu}, \\
\mathcal{B}_\mu(x, \theta, \bar{\theta}) &= \beta_\mu(x) + \theta O_\mu + \bar{\theta}(\beta_\mu^* + F_\mu - \bar{F}_\mu) + \theta \bar{\theta}(o_\mu - \partial_\mu B_2 + \partial_\mu \bar{B}_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_2 \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_2), \\
\bar{\mathcal{B}}_\mu(x, \theta, \bar{\theta}) &= \bar{\beta}_\mu(x) + \theta(\tilde{N}_{\mu\nu} - (\partial_\mu f_\nu - \partial_\mu \bar{f}_\nu - \partial_\nu f_\mu + \partial_\nu \bar{f}_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu f_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{f}_\nu \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu f_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \bar{f}_\mu)) + \bar{\theta} \tilde{B}_{\mu\nu}^* + \theta \bar{\theta} \tilde{n}_{\mu\nu}, \\
\bar{\mathcal{B}}_\mu(x, \theta, \bar{\theta}) &= \bar{\beta}_\mu(x) + \theta(O_\mu - \partial_\mu c_2 + \partial_\mu \bar{c}_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu c_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{c}_2) + \bar{\theta} \beta_\mu^* + \theta \bar{\theta} o_\mu,
\end{aligned}$$

$$\begin{aligned}
\tilde{B}_\mu(x, \theta, \bar{\theta}) &= \tilde{\beta}_\mu(x) + \theta \tilde{O}_\mu + \bar{\theta}(\tilde{\beta}_\mu^\star + \partial_\mu \tilde{c}_2 - \partial_\mu \tilde{\bar{c}}_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{c}_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \tilde{\bar{c}}_2) \\
&\quad + \theta \bar{\theta}(\bar{o}_\mu + \partial_\mu B_2 - \partial_\mu \bar{B}_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_2), \\
\bar{\tilde{B}}_\mu(x, \theta, \bar{\theta}) &= \bar{\tilde{\beta}}_\mu(x) + \theta(\tilde{O}_\mu - \tilde{F}_\mu + \tilde{\bar{F}}_\mu) + \bar{\theta} \tilde{\beta}_\mu^\star + \theta \bar{\theta} \bar{o}_\mu, \\
\mathcal{F}_\mu(x, \theta, \bar{\theta}) &= F_\mu(x) + \theta P_\mu + \bar{\theta} F_\mu^\star + \theta \bar{\theta} p_\mu, \\
\bar{\mathcal{F}}_\mu(x, \theta, \bar{\theta}) &= \bar{F}_\mu(x) + \theta(P_\mu + \partial_\mu B - \partial_\mu \bar{B} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}) + \bar{\theta} F_\mu^\star + \theta \bar{\theta} p_\mu, \\
\tilde{\mathcal{F}}_\mu(x, \theta, \bar{\theta}) &= \tilde{F}_\mu(x) + \theta \tilde{P}_\mu + \bar{\theta}(\tilde{F}_\mu^\star - \partial_\mu B_2 + \partial_\mu \bar{B}_2 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_2) + \theta \bar{\theta} \tilde{p}_\mu, \\
\bar{\tilde{\mathcal{F}}}_\mu(x, \theta, \bar{\theta}) &= \bar{\tilde{F}}_\mu(x) + \theta \tilde{P}_\mu + \bar{\theta} \tilde{F}_\mu^\star + \theta \bar{\theta} \tilde{p}_\mu, \\
\mathfrak{F}_\mu(x, \theta, \bar{\theta}) &= f_\mu(x) + \theta Q_\mu + \bar{\theta}(f_\mu^\star - \partial_\mu B_1 + \partial_\mu \bar{B}_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_1 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_1) + \theta \bar{\theta} q_\mu, \\
\bar{\mathfrak{F}}_\mu(x, \theta, \bar{\theta}) &= \bar{f}_\mu(x) + \theta Q_\mu + \bar{\theta} f_\mu^\star + \theta \bar{\theta} q_\mu, \\
\tilde{\mathfrak{F}}_\mu(x, \theta, \bar{\theta}) &= \tilde{f}_\mu(x) + \theta \tilde{Q}_\mu + \bar{\theta} \tilde{f}_\mu^\star + \theta \bar{\theta} \tilde{q}_\mu, \\
\bar{\tilde{\mathfrak{F}}}_\mu(x, \theta, \bar{\theta}) &= \bar{\tilde{f}}_\mu(x) + \theta(\tilde{Q}_\mu - \partial_\mu B_1 + \partial_\mu \bar{B}_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_1 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_1) + \bar{\theta} \tilde{f}_\mu^\star + \theta \bar{\theta} \tilde{q}_\mu, \\
\mathcal{C}_2(x, \theta, \bar{\theta}) &= c_2(x) + \theta \mathfrak{R} + \bar{\theta}(c_2^\star + B - \bar{B}) + \theta \bar{\theta} r, \\
\bar{\mathcal{C}}_2(x, \theta, \bar{\theta}) &= \bar{c}_2(x) + \theta \mathfrak{R} + \bar{\theta} c_2^\star + \theta \bar{\theta} r, \\
\tilde{\mathcal{C}}_2(x, \theta, \bar{\theta}) &= \tilde{c}_2(x) + \theta \tilde{\mathfrak{R}} + \bar{\theta} \tilde{c}_2^\star + \theta \bar{\theta} \tilde{r}, \\
\bar{\tilde{\mathcal{C}}}_2(x, \theta, \bar{\theta}) &= \bar{\tilde{c}}_2(x) + \theta(\tilde{\mathfrak{R}} - B_2 + \bar{B}_2) + \bar{\theta} \tilde{c}_2^\star + \theta \bar{\theta} \tilde{r}, \\
\mathcal{C}_1(x, \theta, \bar{\theta}) &= c_1(x) + \theta \mathfrak{S} + \bar{\theta}(c_1^\star - B_1 + \bar{B}_1) + \theta \bar{\theta} s, \\
\bar{\mathcal{C}}_1(x, \theta, \bar{\theta}) &= \bar{c}_1(x) + \theta(\mathfrak{S} + B - \bar{B}) + \bar{\theta} c_1^\star + \theta \bar{\theta} s, \\
\tilde{\mathcal{C}}_1(x, \theta, \bar{\theta}) &= \tilde{c}_1(x) + \theta \tilde{\mathfrak{S}} + \bar{\theta}(\tilde{c}_1^\star - B_2 + \bar{B}_2) + \theta \bar{\theta} \tilde{s}, \\
\bar{\tilde{\mathcal{C}}}_1(x, \theta, \bar{\theta}) &= \bar{\tilde{c}}_1(x) + \theta(\tilde{\mathfrak{S}} - B_1 + \bar{B}_1) + \bar{\theta} \tilde{c}_1^\star + \theta \bar{\theta} \tilde{s}, \\
\Phi_\mu(x, \theta, \bar{\theta}) &= \phi_\mu(x) + \theta T_\mu + \bar{\theta}(\phi_\mu^\star + \tilde{f}_\mu - \bar{\tilde{f}}_\mu) + \theta \bar{\theta}(t_\mu + \partial_\mu B_1 - \partial_\mu \bar{B}_1 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu B_1 \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \bar{B}_1), \\
\bar{\Phi}_\mu(x, \theta, \bar{\theta}) &= \bar{\phi}_\mu(x) + \theta(T_\mu - f_\mu + \bar{f}_\mu) + \bar{\theta} \phi_\mu^\star + \theta \bar{\theta} t_\mu, \\
\mathcal{B}(x, \theta, \bar{\theta}) &= B(x) + \theta \mathbb{U} + \bar{\theta} B^\star + \theta \bar{\theta} u, \\
\bar{\mathcal{B}}(x, \theta, \bar{\theta}) &= \bar{B}(x) + \theta \mathbb{U} + \bar{\theta} B^\star + \theta \bar{\theta} u, \\
\mathcal{B}_1(x, \theta, \bar{\theta}) &= B_1(x) + \theta \mathfrak{V} + \bar{\theta} B_1^\star + \theta \bar{\theta} v, \\
\bar{\mathcal{B}}_1(x, \theta, \bar{\theta}) &= \bar{B}_1(x) + \theta \mathfrak{V} + \bar{\theta} B_1^\star + \theta \bar{\theta} v, \\
\mathcal{B}_2(x, \theta, \bar{\theta}) &= B_2(x) + \theta \mathfrak{W} + \bar{\theta} B_2^\star + \theta \bar{\theta} w, \\
\bar{\mathcal{B}}_2(x, \theta, \bar{\theta}) &= \bar{B}_2(x) + \theta \mathfrak{W} + \bar{\theta} B_2^\star + \theta \bar{\theta} w.
\end{aligned} \tag{B.5}$$

Form the above relations, we calculate

$$\begin{aligned}
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{B}}_{\mu\nu\eta} \bar{\mathcal{B}}^{\mu\nu\eta} &= l_{\mu\nu\eta} \bar{B}^{\mu\nu\eta} - B_{\mu\nu\eta}^* (L^{\mu\nu\eta} - \partial^\mu c^{\nu\eta} + \partial^\mu \bar{c}^{\nu\eta} - \partial^\nu c^{\eta\mu} + \partial^\nu \bar{c}^{\eta\mu} - \partial^\eta c^{\mu\nu} + \partial^\eta \bar{c}^{\mu\nu} \\
&\quad + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c^{\nu\eta} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{c}^{\nu\eta} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu c^{\eta\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{c}^{\eta\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\eta c^{\mu\nu} \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\eta \bar{c}^{\mu\nu}), \\
\frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{C}}_{\mu\nu} \bar{\mathcal{C}}^{\mu\nu} &= \bar{m}_{\mu\nu} \bar{c}^{\mu\nu} + m_{\mu\nu} \bar{c}^{\mu\nu} + \bar{c}_{\mu\nu}^* (M^{\mu\nu} - \partial^\mu \beta^\nu + \partial^\mu \bar{\beta}^\nu + \partial^\nu \beta^\mu - \partial^\nu \bar{\beta}^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \beta^\nu \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{\beta}^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \beta^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{\beta}^\mu) + c_{\mu\nu}^* (\tilde{M}^{\mu\nu} - B^{\mu\nu} + \bar{B}^{\mu\nu}), \\
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{B}}_{\mu\nu} \bar{\mathcal{B}}^{\mu\nu} &= n_{\mu\nu} \bar{B}^{\mu\nu} - B_{\mu\nu}^* N^{\mu\nu}, \\
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{B}}_{\mu\nu} \bar{\mathcal{B}}^{\mu\nu} &= \bar{n}_{\mu\nu} \bar{\tilde{B}}^{\mu\nu} - \tilde{B}_{\mu\nu}^* (\tilde{N}^{\mu\nu} - \partial^\mu f^\nu + \partial^\mu \bar{f}^\nu + \partial^\nu f^\mu - \partial^\nu \bar{f}^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu f^\nu \\
&\quad - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{f}^\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu f^\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu \bar{f}^\mu), \\
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{B}}_\mu \bar{\mathcal{B}}^\mu &= o_\mu \bar{\beta}^\mu - \beta_\mu^* (O^\mu - \partial^\mu c_2 + \partial^\mu \bar{c}_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu c_2 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{c}_2), \\
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{B}}_\mu \bar{\mathcal{B}}^\mu &= \bar{o}_\mu \bar{\tilde{\beta}}^\mu - \tilde{\beta}_\mu^* (\tilde{O}^\mu - \tilde{F}^\mu + \bar{\tilde{F}}^\mu), \\
\frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{F}}_\mu \bar{\mathcal{F}}^\mu &= \bar{p}_\mu \bar{F}^\mu + p_\mu \bar{\tilde{F}}^\mu + \tilde{F}_\mu^* (P^\mu + \partial^\mu B - \partial^\mu \bar{B} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{B}) + F_\mu^* \tilde{P}^\mu, \\
\frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{F}}_\mu \bar{\mathcal{F}}^\mu &= \bar{q}_\mu \bar{f}^\mu + q_\mu \bar{\tilde{f}}^\mu + \tilde{f}_\mu^* Q^\mu + f_\mu^* (\tilde{Q}^\mu - \partial^\mu B_1 + \partial^\mu \bar{B}_1 + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B_1 - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \bar{B}_1), \\
\frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{C}}_2 \bar{\mathcal{C}}_2 &= \bar{r} \bar{c}_2 + r \bar{\tilde{c}}_2 + \tilde{c}_2^* \mathfrak{R} + c_2^* (\mathfrak{R} - B_2 + \bar{B}_2), \\
\frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{C}}_1 \bar{\mathcal{C}}_1 &= \bar{s} \bar{c}_1 + \bar{c}_1^* (\mathfrak{S} + B - \bar{B}) + s \bar{\tilde{c}}_1 + c_1^* (\tilde{\mathfrak{S}} - B_1 + \bar{B}_1), \\
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\Phi}_\mu \bar{\Phi}^\mu &= t_\mu \bar{\phi}^\mu - \phi_\mu^* (T^\mu - f^\mu + \bar{f}^\mu), \\
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{B}} \bar{\mathcal{B}} &= u \bar{B} - B^* \mathfrak{U}, \\
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{B}}_1 \bar{\mathcal{B}}_1 &= v \bar{B}_1 - B_1^* \mathfrak{V}, \\
\frac{1}{2} \frac{\delta}{\delta \theta} \frac{\delta}{\delta \theta} \bar{\mathcal{B}}_2 \bar{\mathcal{B}}_2 &= w \bar{B}_2 - B_2^* \mathfrak{W}.
\end{aligned} \tag{B.6}$$

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